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DYNAMIC DATA ANALYSIS TECHNIQUES USED IN THE LANGLEY TIME SERIES ANALYSIS COMPUTER PROGRAM

by Robert C. Ward

Langley Research Center
Hampton, Va. 23365

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ERRATA

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Page 2: Third line from bottom of page should read

B_{pc} confidence band for spectral estimators

Page 34:

Third, fourth, fifth, and sixth lines under the heading "Accuracy Test" should be replaced by

confidence percentage, the program computes an interval spread in decibels for the spectral estimations. For further information, see Blackman and Tukey (ref. 3, sec. 9, pp. 21-25). The equation for the interval spread is as follows:

The line just above the heading "Off-Scale Test" should read

S_{db} chi-square distribution parameter for specified confidence percentage (see table VIII)

Page 50:

Title of table VIII should read

CHI-SQUARE DISTRIBUTION PARAMETER FOR SPECIFIED CONFIDENCE PERCENTAGES

Also, the heading for the first column in this table should be changed from "Decibel spread" to "Chi-square parameter"

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16. Abstract

This paper is intended to be a reference guide to users of the Langley time series analysis computer program (designated R1299), which is a general purpose program for the analysis of random, stationary time series. The bases for choosing available options, descriptions of the options, possible output quantities, equations of the various computations, and tables and figures which might aid in the analysis of data are included.

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SUMMARY

This paper is intended to be a reference guide to users of the Langley time series analysis program (designated R1299). The bases for choosing available options, descriptions of the options, possible output quantities, equations of the various computations, and useful tables and figures which might aid in the analysis of data are included.

INTRODUCTION

The Langley digital time series analysis (TSA) program is a general purpose program for the analysis of random, stationary time series, and the relationships between these series. The major output quantities are probability density estimators (amplitude domain), correlation estimators (time domain), and spectral estimators (frequency domain). A choice of standard (histograms for probability densities; correlations and Fourier transforms for spectra) or novel techniques (orthogonal functions for probability densities; recursive filtering for spectra) may be specified.

Up to 20 different channels of data may be processed at one time. The input data may be conditioned by removal of its mean, corrected for a different bias in each channel, and subjected to a generalized preconditioning (shaping) filter with appropriate post-darkening of the spectra output. The preconditioning filter allows an arbitrary cascade of first-difference, high-pass, low-pass, band-pass, and band-reject filters.

The actual processing of the conditioned data may include the computation of autocovariances and cross-covariances; autocorrelations and cross-correlations; autospectral and cross-spectral densities; average frequency, equivalent bandwidth, and equivalent duration in each spectral band; coherences; interpowers; transfer functions; filter response functions; time to maximum cross-correlation; spectral confidence bands; means; standard deviations; normalized third and fourth central moments; normality tests; histograms; and orthogonal coefficients for probability distribution.

Since the TSA program is the major analytical tool for the analysis of dynamic data at Langley Research Center, a reference manual would be of considerable benefit to users or prospective users of this program. The purpose of this paper is to supply the user with this reference manual. (One should be aware of the different definitions placed on such terms as autocovariance and autocorrelation which differ from one book to the next. The defining equations should always be checked before comparing results from two references.)

Some of the information found in this paper was obtained from reference 1, which is the documentation of the TSA parent program written under NASA Contract No. NAS 1-5632 and the information is included in this document for completeness.

SYMBOLS

$A_{fo}^{a}(f')$	filter response function for analysis filters
A_j	resultant nonrecursive weight
$A_j^{(i)}$	cascaded nonrecursive weight for ith precondition filter
$A_o^I(f_o)$	quadrature filter nonrecursive weight
$A_o^R(f_o)$	in-phase filter nonrecursive weight
a_j	coefficients for probability density function expansion
$a_k^{(i)}$	nonrecursive weight for ith precondition filter in cascade
B_i	resultant recursive weight; bandwidth for frequencies in filtering technique arbitrary analysis
$B_i^I(f_o)$	quadrature filter recursive weight
$B_i^R(f_o)$	in-phase filter recursive weight
$B_m^{(i)}$	cascaded recursive weight for ith precondition filter
B_{pc}	percentage confidence band
$B_x(f_o)$	equivalent bandwidth for channel x
b	an endpoint of interval containing data samples

$b_i(f_o)$	recursive weight to cascade for analysis filters
$b_k^{(i)}$	recursive weight for ith precondition filter in cascade
$C_{xy}(f)$	real part of cross-spectral density between channels x and y
$C_{xy}^N(f)$	real part of normalized cross-spectral density between channels x and y
D	resultant number of recursive weights for precondition filter
$D(f_o)$	decimation factor for analysis filter
D_i	number of recursive weights for ith filter in cascade
$D_x(f_o)$	equivalent duration for channel x
d_i	number of recursive weights for precondition filter
d_p	precondition decimation factor
$F_j(z_i)$	Hermite or Laguerre orthonormal functions
f	frequency
f'	normalized frequency, f/f_s
Δf	frequency resolution for power spectra in correlation technique; filter bandwidth in filtering technique equispaced analysis
f_B	filter bandwidth
f'_B	normalized filter bandwidth
f'_c	normalized precondition filter cutoff frequency
f_I	maximum frequency of interest in analog data
f_i	ith frequency to be analyzed in filtering technique

f_{max}	maximum frequency to be analyzed in filtering technique
f_N	Nyquist frequency, $f_s/2$
f_o	filter center frequency
f'_o	normalized filter center frequency, f_o/f_s
f^*_o	center frequency of filter preceding f_o
f_s	sampling frequency
$\bar{f}_x(f_o)$	average frequency for channel x
$G_x(f), G_y(f)$	power spectral density for channels x and y
$G_x^N(f)$	normalized power spectral density for channel x
$G_{xy}(f)$	amplitude of cross-spectral density between channels x and y
$H(f)$	instrumentation system amplitude response
$H^a_{fo}(f')$	complex filter response
$H_x(f), H_y(f)$	amplitude frequency response for channels x and y from calibration tables
$I(x)$	integer part of x
$I_n^x(f_o), I_n^y(f_o)$	nth data point for channels x and y, respectively, filtered through quadrature filter with f_o center frequency
I_{xy}	interpower between channels x and y
K	fourth central moment (kurtosis); number of degrees of freedom
k	index; number of filters per octave in filtering technique constant Q analysis
M	number of filtered data points after decimation

m	number of lags in correlation technique
N	number of data samples to analyze; resultant number of nonrecursive weights
N_i	number of nonrecursive weights minus one for ith filter in cascade; number of points in ith histogram bin
N_w	number of filter warmup samples
n_i	number of nonrecursive weights minus one
P	number of expansion terms in probability expansion
$P(x)$	smooth probability density function
$P(z)$	smooth normalized probability density function
P_i	probability of being in ith histogram bin if Gaussian
$Q_{xy}(f)$	imaginary part of cross-spectral density between channels x and y
$Q_{xy}^N(f)$	imaginary part of normalized cross-spectral density between channels x and y
$R_x(\tau),R_y(\tau)$	autocovariances of channels x and y, respectively
$R_{xy}(\tau)$	cross-covariance between channels x and y
$R_n^x(f_o),R_n^y(f_o)$	nth data point for channels x and y, respectively, filtered through in-phase filter with f_o center frequency
r	harmonic number in power spectral density equation
S_{db}	decibel spread for percentage confidence band
T	time length of data to be digitized
$T(f)$	precondition filter transfer function
$T_{xy}(f)$	amplitude transfer function between channels x and y

Δt	sampling interval (time increment)
V	number of analysis filter weights
V_P	number of filters in cascade
$\overline{X}$	mean value
$\hat{X}$	bias for channel x
$\tilde{X}_i$	input data for channel x with bias subtracted
X_i	data sample with or without the mean and/or bias subtracted
X_i'	preconditioned data sample with or without the mean and/or bias subtracted
x,y	data channels
x_i	data sample for channel x
y_i	data sample for channel y
z_i	normalized data sample
α	integration variable
β	third central moment (skewness)
$\beta(f)$	instrumentation system phase response
$\beta(f_o)$	noise bandwidth of analysis filter pair
$\beta_x(f), \beta_y(f)$	phase response for channels x and y, respectively, from calibration tables
γ	intermediate variable; test for normality value
$\gamma_{xy}(f)$	coherence between channels x and y
λ_k	"lag window" (smoothing parameter) defined in power spectral density equation

λ_o	effective filter center frequency after decimation
$\rho_{xy}(\tau)$	cross-correlation between channels x and y
$\rho_x(\tau)$	autocorrelation of channel x
σ	standard deviation
τ	lag time (displacement)
τ_{xy}	time lag between channels x and y
τ_{xy}^{ρ}	time lag to maximum cross-correlation between channels x and y
$\Phi_{fo}^{a}(f')$	analysis filter phase response
$\Delta\Phi_{fo}(f')$	analysis filter pair phase response difference
$\Phi_{xy}(f)$	phase of cross-spectral density between channels x and y
$\hat{\Phi}_{xy}(f)$	phase of normalized cross-spectral density between channels x and y
$\chi^2_{K,\alpha}$	chi-square value for K degrees of freedom and α type 1 error

Subscripts:

imag	imaginary
real	real
max	maximum

DYNAMIC DATA REDUCTION

The general procedure for reducing dynamic data in recorded analog form is found in figure 1. The solid lines represent required steps and the dashed lines represent optional steps and output. The digitizing process obtains a data sample every Δt seconds where Δt is determined by the sampling frequency as discussed later. The units of the digitized data are normalized counts. The quantity pass program converts the data units from normalized counts to the physical units as recorded. The time-history plot and list program produces a listing of all or part of the input data and/or a "calcomp" plot of the data. The TSA output calcomp plot program produces calcomp plots of the TSA output functions. Documentation and information concerning these programs are available upon request from the Langley Analysis and Computation Division.

If the data are already in digital form, the only step preceding the analysis performed by the TSA program is a format change. The data must be changed to the format accepted by the TSA program as shown in table I. If at all possible, the data should also be blocked as outlined in table II, although provisions are made to accept either blocked or unblocked data. An example of a FORTRAN WRITE statement for producing an unblocked data tape with six data channels is as follows:

```
WRITE (1) (DATA(I),I=1, 16).
```

DATA (3) through DATA (8) should have been equivalenced to a fixed-point array (dimension 6) and the Hollerith engineering identification entered in this array. An example of a FORTRAN WRITE statement for producing a blocked data tape with six data channels is as follows:

```
WRITE (1) (DATA(I),I=1, 500).
```

DATA (1) through DATA (16), DATA (21) through DATA (36), etc., correspond to this example of the unblocked data array.

Figure 2 shows a simplified flow of data through the three major sections of the TSA program. These sections are the probability density estimation, preconditioning of the data, and the spectral estimation. Each of these blocks as well as the computation method within each block is independent of the others. Thus, the user may choose not to compute the probability density function or choose either computation method desired without affecting his decision concerning computation of the spectral estimators or preconditioning of the data.

SAMPLING FREQUENCY FOR DIGITIZING

The sampling frequency f_s at which the data is digitized is a very important factor in the dynamic data reduction process. This frequency determines the Nyquist frequency f_N, which is the highest frequency that can be analyzed, by the following relationship:

$$f_N = \frac{1}{2} f_s \tag{1}$$

Therefore, one could determine the necessary sampling frequency in theory by the equation $f_S = 2f_I$ where f_I is the maximum frequency of interest contained in the data. With f_S determined in this manner, there are only two points per cycle for determining the power at this maximum frequency of interest. Although two points per cycle of the frequency f_I is the theoretical requirement, more points are recommended in practice for improved results. The suggested procedure at Langley Research Center is to select f_S as follows:

$$f_S = 5f_I \tag{2}$$

Central data transcription at Langley Research Center automatically filters the data with an analog low pass filter before the data are digitized. This filter eliminates the problem of folding known as "aliasing." (See ref. 2, pp. 279-281.)

In the remaining part of this paper, input data refer to the data after it has been digitized, converted to physical units, and recorded in the format shown in table I.

TREATMENT OF THE INPUT DATA

Several options are available for calibration and preconditioning of the input data. These options may be chosen independently of each other and of the methods chosen for the actual processing. The options are listed and the governing equations are given.

Bias Calibration

A bias level (zero level) value may be entered to subtract from each channel. The bias is subtracted before any computations are performed on the data; therefore, all output quantities reflect the data with the bias subtracted. If a bias is not entered for a particular channel, the bias to be subtracted for that channel equals zero.

$$\tilde{X}_i = x_i - \hat{X} \tag{3}$$

where

x_i input data for channel x

$\hat{X}$ bias for channel x

Mean Removal

By the proper selection of an input parameter, the mean may be removed from each channel before the probability and spectral estimators are computed. The mean is computed after the bias, if any, is subtracted from each data point.

If the mean is not removed, it will show up in the power spectral density function as a delta function of mean squared magnitude at zero frequency. Since the delta function theoretically is infinite and if there were any frequency components in the data near zero, one would not be able to determine them because they would be completely masked by the delta function. In actual practice, the delta function would just be a very sharp peak near zero frequency and would have a finite bandwidth. This condition is one of the reasons why the mean, in the general case, should be removed.

$$X_i = \tilde{X}_i - \overline{X} \tag{4}$$

where

$\tilde{X}_i$ input data for channel x with bias subtracted

$\overline{X}$ mean value of channel x

Preconditioning With a Shaping Filter

Spectral computations at low power level are considerably distorted by the presence of high power in adjacent frequency bands. These distortions are reduced by the technique of preconditioning the data by a shaping filter which tends to flatten the spectra or make it look more like white noise. (The effect of this filter may then be removed from the output by dividing the spectra by the filter transfer function. This procedure is called postdarkening and is discussed later under "Modification of Spectral Output.") The shaping filters may also be used to remove undesirable frequencies from the data before analysis.

The shaping filter design allows any of the following different types of simple filters: first difference (an approximation to the first derivative filter), low pass, high pass, band pass, and band reject. Also, it allows an arbitrary series combination (cascading) of the preceding types. A filter made up of a series combination of simple filters is called a compound filter. A preconditioning restriction is that all channels must be treated with the same filter.

Figure 3 shows some typical transfer functions of the simple filters. Figure 4 shows two examples of cascaded compound shaping filters. These transfer functions have an abscissa of normalized frequency f'. Normalized frequency is the frequency divided by the sampling frequency.

The filters are designed by use of a recursive technique. This technique is desirable since it requires a smaller amount of computer storage and less time to perform the desired function. A recursive filter is a filter which has a feedback of the output samples into the filter. This filter is illustrated in figure 5 which shows both the pictoral and mathematical representation. The two summations in the mathematical equation are the nonrecursive summation with the nonrecursive weights A_j and the recursive summation with the recursive weights B_m. The type of shaping filter designed is completely determined by A_j, B_m, the number of nonrecursive weights N, and the number of recursive weights D. A note of caution in specifying these filters is that the recursive filter takes a time approximately equal to the reciprocal of the filter bandwidth to stabilize. If this time is more than 10 percent of the data span, the spectral estimators may be adversely affected.

Preconditioning (if requested) is implemented by a shaping filter containing nonrecursive weights A_j and recursive weights B_m. If a compound filter is requested, these weights are computed by cascading the weights of the simple filters. The ith simple filter has $n_i + 1$ nonrecursive weights $a_0^{(i)}, a_1^{(i)}, a_2^{(i)}, \ldots, a_{n_i}^{(i)}$ and d_i recursive weights $b_1^{(i)}, b_2^{(i)}, \ldots, b_{d_i}^{(i)}$.

The various simple filter classes have the following synthesis:

First difference:

$$\left.\begin{aligned} n_i &= 1 \\ d_i &= 0 \\ a_0^{(i)} &= 0.5 \\ a_1^{(i)} &= -0.5 \end{aligned}\right\} \tag{5}$$

Low pass (for cascading):

$$\left.\begin{aligned} n_i &= 0 \\ d_i &= 1 \\ b_1^{(i)} &= \exp\left(-2\pi f'_c\right) \\ a_0^{(i)} &= 1 - b_1^{(i)} \end{aligned}\right\} \tag{6}$$

Low pass (for simple filter):

$$\left.\begin{aligned} n_1 &= 1 \\ d_1 &= 1 \\ a_1^{(1)} &= 0.5\left[1 + \tan\left(\pi f'_c\right) - \sec\left(\pi f'_c\right)\right] \\ a_2^{(1)} &= a_1^{(1)} \\ b_1^{(1)} &= 1 - 2a_1^{(1)} \end{aligned}\right\} \tag{7}$$

High pass (for cascading):

$$\left.\begin{aligned} n_i &= 0 \\ d_i &= 1 \\ b_1^{(i)} &= -\exp\left(-2\pi f'_c\right) \\ a_0^{(i)} &= 1 + b_1^{(i)} \end{aligned}\right\} \tag{8}$$

High pass (for simple filter):

$$\left.\begin{aligned} n_1 &= 1 \\ d_1 &= 1 \\ a_1^{(1)} &= 0.5\left[1 + \sec\left(\pi f'_c\right) - \tan\left(\pi f'_c\right)\right] \\ a_2^{(1)} &= -a_1^{(1)} \\ b_1^{(1)} &= 2a_1^{(1)} - 1 \end{aligned}\right\} \tag{9}$$

Band pass:

$$\left.\begin{aligned} n_i &= 0 \\ d_i &= 2 \\ b_2^{(i)} &= -\exp\left(-2\pi f'_B\right) \\ b_1^{(i)} &= -2b_2^{(i)} \cos\left(2\pi f'_o\right) \\ a_0^{(i)} &= \Big[1 + \left(b_1^{(i)}\right)^2 + \left(b_2^{(i)}\right)^2 - 2b_1^{(i)} \cos(2\pi f') - 2b_2^{(i)} \cos(4\pi f') \\ &\qquad + 2b_1^{(i)} b_2^{(i)} \cos\left(2\pi f'_o\right)\Big]^{1/2} \end{aligned}\right\} \tag{10}$$

This filter is a band-pass filter with zero phase at the normalized center frequency f'_o and the maximum response slightly greater than 1.

Band reject:

$$
\left.
\begin{aligned}
n_i &= 2 \\
d_i &= 2 \\
b_2^{(i)} &= -\exp\left(-2\pi f'_B\right) \\
b_1^{(i)} &= -2b_2^{(i)} \cos\left(2\pi f'_o\right) \\
a_2^{(i)} &= -b_2^{(i)} \\
a_1^{(i)} &= -b_1^{(i)} \\
a_0^{(i)} &= 1 - \left[1 + \left(b_1^{(i)}\right)^2 + \left(b_2^{(i)}\right)^2 - 2b_1^{(i)} \cos(2\pi f') - 2b_2^{(i)} \cos\left(4\pi f'_o\right)\right. \\
&\qquad \left. + 2b_1^{(i)} b_2^{(i)} \cos(2\pi f')\right]^{1/2}
\end{aligned}
\right\} \tag{11}
$$

The weights for the ith cascaded filter are built up from the (i - 1)th filter by the following recurrence relationships:

$$A_j^{(i)} = \sum_{k=0}^{n_i} a_k^{(i)} A_{j-k}^{(i-1)} \qquad (j = 0, 1, \ldots, N_i) \tag{12}$$

$$B_m^{(i)} = B_m^{(i-1)} - \sum_{k=1}^{d_i} b_k^{(i)} B_{m-k}^{(i-1)} \qquad (m = 1, 2, \ldots, D_i) \tag{13}$$

where

$$N_i = N_{i-1} + n_i$$

$$D_i = D_{i-1} + d_i$$

with the initial conditions:

$$A_0^{(0)} = 1$$

$$A_j^{(0)} = 0 \qquad (j \neq 0)$$

$$B_0^{(0)} = -1$$

$$B_m^{(0)} = 0 \qquad (m \neq 0)$$

When X_i is used to denote the ith data sample with or without the mean and/or bias subtracted, the application of the shaping filter is then:

$$X_i' = \sum_{j=0}^{N} A_j X_{i-j} + \sum_{m=1}^{D} B_m X_{i-m}' \tag{14}$$

If the spectra are to be computed by the filtering technique and $f_{max}\,\Delta t < \frac{1}{5}$, the shaping filter output is decimated by d_p which is determined by the following equation:

$$d_p = I\left(\frac{1}{5f_{max}\,\Delta t}\right) \tag{15}$$

where

Δt time increment

$I(x)$ integer part of x

f_{max} maximum frequency to analyze in filtering technique

If preconditioning is requested, all spectral answers are computed by using X_i' and all probabilities are computed by using X_i. In the remaining equations, x_i will be used for X_i or X_i' depending on the treatment of the data.

SPECTRAL METHODS

There are two types of spectral computations available for use in the program. They are (1) the traditional correlation-Fourier transform techniques and (2) a novel filtering technique that is directly analogous to an analog filter bank.

The correlation-Fourier transform method is to be preferred when any of the following occurs:

(1) Autocorrelation or cross-correlation functions are required for data reduction.

(2) Cross-spectra are required on a small number of channels (small as compared with the number of channels recorded).

(3) Spectral analyses are desired at a constant frequency resolution from zero frequency to the Nyquist frequency (half the sampling rate).

(4) The filter warmup time cannot be tolerated.

(5) The input data contain a large number of data samples and channels (large is on the order of 20 000 samples and 8 channels).

The filtering method is to be preferred when:

(1) Correlation functions are not required for data reduction.

(2) Cross-spectra are required on a large number of channels.

(3) Spectral analyses are desired at a nonuniform frequency spacing (such as constant Q analysis or a high resolution analysis over a small section of the spectrum).

(4) Auxiliary information (average frequency, equivalent bandwidth and equivalent duration in each frequency band) is required for stationary or randomness indicators.

(5) Spectral answers are not desired at the Nyquist frequency and zero frequency.

An additional difference in the spectral output between the two techniques is that the leakage effects due to side lobes of the estimation process in frequency can result in only positive power in the filtering approach, but in positive or negative power by the correlation technique. Therefore, leakage from adjacent frequencies can result in negative coherences or high coherences (greater than 1) at low power levels in the correlation analysis, whereas the coherence function from the filtering analysis is always between 0 and 1. However, the filter approach may give a total integrated power that is greater than the total variance whereas the correlation method always insures that the integrated power is the total variance.

Unless the user is reasonably certain about the frequency range of interest of his data, it is suggested that the correlation technique be used. This technique gives the user the maximum amount of information concerning the frequency content of his data. Thus, all sampled frequencies would be displayed, including any unexpected frequencies. This method uncovers possible errors in recording or digitizing the data. Also, the user may then specify certain frequency ranges for detailed analysis by using the filtering technique.

Correlation Technique

The correlation method also known as the Blackman-Tukey method is used to compute the spectral estimators. For a fixed data length T, the only spectral parameter not specified is the number of lags.

The number of lags m is given by the following equation:

$$m = \frac{1}{2\ \Delta t\ \Delta f} \tag{16}$$

where Δf is the desired equivalent resolution bandwidth (frequency resolution) for power spectral calculations. The term Δf may have to be slightly altered so that m will be an integer. The maximum number of lags allowed by the TSA program is 999. See tables III and IV for the maximum number of channels to process at one time based on the number of lags. In order to give good statistical reliability, it is desirable to keep the maximum number of lags m less than one-tenth the sample size. The sample size N is determined by T (time length of the data) and Δt by the relationship

$$N = \frac{T}{\Delta t} \tag{17}$$

The Blackman-Tukey method of computing the spectral estimators consists basically of computing the covariance function and performing a Fourier transform of this function. (See appendix A for the general computer procedure.) To eliminate the effect of taking the transform of a truncated function, the covariance function (truncated in time at $m\,\Delta t$) is multiplied by a "lag window" as the transform is performed. This multiplication smooths the power spectral density, as explained in reference 3 (pp. 11 to 15).

It should be noted that the TSA program does not distinguish between functions obtained from a time series with the mean subtracted and those obtained from a time series without the mean subtracted. Thus, one should exercise care when referring to other references on dynamic data about specific functions. For example, in the time domain functions, reference 2 denotes the function obtained from a time series with the mean subtracted as "autocovariance," the function obtained from a time series without the mean subtracted or with a mean of zero as "autocorrelation," and the normalized function with the mean subtracted as "Correlation coefficient."

The equations for these functions and associated functions used by the TSA program are as follows:

For autocovariance,

$$R_X(k\,\Delta t) = \frac{1}{N - k} \sum_{i=1}^{N-k} x_i x_{i+k} \qquad (k = 0, 1, 2, \ldots, m) \tag{18}$$

where

k	the lag number
m	maximum lag number
x	channel
$k\,\Delta t$	displacement, τ
N	number of data points

For autocorrelation,

$$\rho_x(k\,\Delta t) = \frac{R_x(k\,\Delta t)}{R_x(0)} \qquad (k = 0, 1, 2, \ldots, m) \quad (19)$$

For cross-covariance,

$$R_{xy}(k\,\Delta t) = \frac{1}{N - k} \sum_{i=1}^{N-k} x_i y_{i+k} \qquad (k = 0, 1, 2, \ldots, m) \quad (20)$$

$$R_{xy}(-k\,\Delta t) = R_{yx}(k\,\Delta t) = \frac{1}{N - k} \sum_{i=1}^{N-k} y_i x_{i+k} \qquad (k = 0, 1, 2, \ldots, m) \quad (21)$$

For cross-correlation,

$$\rho_{xy}(\pm k\,\Delta t) = \frac{R_{xy}(\pm k\,\Delta t)}{\left[R_x(0) R_y(0)\right]^{1/2}} \qquad (k = 0, 1, 2, \ldots, m) \quad (22)$$

For power spectral density,

$$G_x\left(\frac{rf_N}{m}\right) = 2\,\Delta t\left[R_x(0) + 2 \sum_{k=1}^{m} \lambda_k R_x(k\,\Delta t) \cos\left(\frac{\pi k r}{m}\right)\right] \qquad (r = 0, 1, 2, \ldots, m) \quad (23)$$

where

r	harmonic number
$\frac{rf_N}{m}$	frequency, f
x	channel
λ_k	"lag window" (smoothing parameter)

For Hamming smoothing, λ_k is defined as

$$\lambda_k = 0.54 + 0.46 \cos \frac{\pi k}{m} \qquad (k = 1, 2, \ldots, m-1) \quad (24a)$$

$$\lambda_k = 0.04 \qquad (k = m) \quad (24b)$$

For Hanning smoothing, λ_k is defined as

$$\lambda_k = 0.50 + 0.50 \cos \frac{\pi k}{m} \qquad (k = 1, 2, \ldots, m-1) \quad (25a)$$

$$\lambda_k = 0 \qquad (k = m) \qquad (25b)$$

For normalized power spectral density,

$$G_x^N\left(\frac{rf_N}{m}\right) = \frac{G_x\left(\frac{rf_N}{m}\right)}{R_x(0)} \qquad (r = 0, 1, 2, \ldots, m) \qquad (26)$$

For cross-spectral density,

$$C_{xy}\left(\frac{rf_N}{m}\right) = 2\,\Delta t\left[D_{xy}(0) + 2\sum_{k=1}^{m} \lambda_k D_{xy}(k\,\Delta t)\cos\left(\frac{\pi kr}{m}\right)\right] \qquad (r = 0, 1, 2, \ldots, m) \qquad (27)$$

$$Q_{xy}\left(\frac{rf_N}{m}\right) = 4\,\Delta t\left[\sum_{k=1}^{m} \lambda_k D'_{xy}(k\,\Delta t)\sin\left(\frac{\pi kr}{m}\right)\right] \qquad (r = 0, 1, 2, \ldots, m) \qquad (28)$$

where

$$D_{xy}(k\,\Delta t) = \frac{1}{2}\left[R_{xy}(k\,\Delta t) + R_{yx}(k\,\Delta t)\right]$$

$$D'_{xy}(k\,\Delta t) = \frac{1}{2}\left[R_{xy}(k\,\Delta t) - R_{yx}(k\,\Delta t)\right]$$

$$G_{xy}(f) = \sqrt{\left[C_{xy}(f)\right]^2 + \left[Q_{xy}(f)\right]^2} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \qquad (29)$$

$$\Phi_{xy}(f) = \text{arc tan}\,\frac{Q_{xy}(f)}{C_{xy}(f)} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \qquad (30)$$

For normalized cross-spectral density,

$$C_{xy}^N(f) = \frac{C_{xy}(f)}{\sqrt{G_x(f)\ G_y(f)}} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \qquad (31)$$

$$Q_{xy}^N(f) = \frac{Q_{xy}(f)}{\sqrt{G_x(f)\ G_y(f)}} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \qquad (32)$$

If $G_x(f)$ or $G_y(f)$ is negative, $C^N_{xy}(f) = Q^N_{xy}(f) = 0$.

$$\hat{\Phi}_{xy}(f) = \Phi_{xy}(f) \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \tag{33}$$

For coherence,

$$\gamma_{xy}(f) = \frac{G_{xy}(f)}{\sqrt{G_x(f)\ G_y(f)}} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \tag{34}$$

For amplitude transfer function

$$T_{xy}(f) = \frac{G_{xy}(f)}{G_x(f)} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \tag{35}$$

$$T_{yx}(f) = \frac{G_{xy}(f)}{G_y(f)} \qquad \left(f = 0, \frac{f_N}{m}, \frac{2f_N}{m}, \ldots, f_N\right) \tag{36}$$

For interpower,

$$I_{xy} = \frac{\sum_{r=0}^{m} G_y\left(\frac{rf_N}{m}\right) \gamma_{xy}\left(\frac{rf_N}{m}\right)}{\sum_{r=0}^{m} G_y\left(\frac{rf_N}{m}\right)} \tag{37}$$

$$I_{yx} = \frac{\sum_{r=0}^{m} G_x\left(\frac{rf_N}{m}\right) \gamma_{xy}\left(\frac{rf_N}{m}\right)}{\sum_{r=0}^{m} G_x\left(\frac{rf_N}{m}\right)} \tag{38}$$

Filtering Technique

The filtering technique involves the design of a band-pass filter for each frequency the user wishes to analyze. The data are passed through each filter and the output is squared and averaged to determine the spectral results. The only spectral parameters necessary for this process are the frequency specifications and the number of filter weights.

The program allows the following specifications for the frequency variable in the filtering approach:

(1) Equispaced analysis: For equispaced analysis the frequencies are from f_1 to f_{max} at equal intervals of Δf. The filters will have bandwidths of Δf and will be centered on f_1, $f_1 + \Delta f$, $f_1 + 2\Delta f$, . . ., and so forth, until $(f_1 + m\,\Delta f) \geqq f_{max}$.

(2) Constant Q analysis: For the constant Q analysis the frequencies are from f_1 to f_{max} at equal logarithmic intervals given as k per octave. The filters will have center frequencies of f_1, $f_1 \cdot 2^{1/k}$, $f_1 \cdot 2^{2/k}$, $f_1 \cdot 2^{3/k}$, . . ., and so forth, and bandwidths of $f_1\left[\left(2^{1/k} - 2^{-1/k}\right)/2\right]$, $f_1\left[\left(2^{2/k} - 1\right)/2\right]$, $f_1\left[\left(2^{3/k} - 2^{1/k}\right)/2\right]$, . . ., and so forth, respectively, until $f_1 2^{m/k} \geqq f_{max}$.

(3) Arbitrary analysis: For the arbitrary analysis, a table of frequencies f_1, f_2, . . ., f_{max} and the associated bandwidths B_1, B_2, . . ., B_{max} are read from the parameter deck.

A restriction on the frequency variable is that f_1 cannot equal 0 and f_{max} cannot equal the Nyquist frequency. See tables V and VI for restrictions on the number of filters generated. Figures 6(a) to 6(c) illustrate each of these band-pass filter options. The plots shown are used for descriptive purposes only and are not response functions of the actual analysis filters.

The analysis filters are cascades of simple band-pass filters. The outputs are computed by the use of recursive equations on the past outputs; therefore, they have an initial settling time when the data are "turned on." The program allows the number of data samples equal to the reciprocal of the normalized filter bandwidth for a warmup period before the filter outputs are used in spectral computations.

For cross-spectral computations, both "in-phase" (real) and "quadrature" (imaginary) filters are required. (See fig. 7.) These filters are produced by including filters in the cascade that shift the phase $+45^{\circ}$ and -45° at the center frequency. The center frequency of the filter is shifted to the low-frequency side for the in-phase and to the high-frequency side for quadrature phase. By setting the proper input parameters, the program will print and/or plot the desired filter response functions.

The program allows the user a choice of the number of filter weights used to construct the required band-pass filters. The user can choose between 5, 7, or 9 weights which corresponds to a cascade of 2, 3, or 4 filters.

Naturally, a sharper and more precise filter can be constructed when using more filters in the cascade, but this procedure requires more computer time and storage.

Therefore, a trade-off must be made in terms of filter characteristics and computer time and storage. See tables V and VI for restrictions on the number of filter weights.

Each of the filters has the following recursive weights:

$$b_1(f_o) = 2\,\exp(-\pi\gamma)\cos\left\{2\pi\omega_o\left[1 - \left(\frac{\gamma}{2\omega_o}\right)^2\right]^{1/2}\right\} \tag{39a}$$

$$b_2(f_o) = -\exp(-2\pi\gamma) \tag{39b}$$

where

$$\gamma = \begin{cases} 0.4f'_o & \text{(For first filter in cascade)} \\ \dfrac{f'_B}{\left(2^{1/V_p} - 1\right)^{1/2}} & \text{(All remaining filters in cascade)} \end{cases}$$

$$\omega_o = \begin{cases} 0.8f'_o & \text{(For first filter in cascade (in-phase))} \\ 1.2f'_o & \text{(For first filter in cascade (quadrature))} \\ f'_o & \text{(All remaining filters in cascade)} \end{cases}$$

f'_B normalized filter bandwidth associated with f_o

V_p number of filters in cascade

For both the in-phase filter recursive weights $B_j^R(f_o)$, and quadrature filter recursive weights $B_j^I(f_o)$, the ith cascaded filter is built up from the $(i - 1)^{th}$ filter by the following relationship:

$$B_j^{(i)}(f_o) = B_j^{(i-1)}(f_o) - b_1(f_o)\,B_{j-1}^{(i-1)}(f_o) - b_2(f_o)\,B_{j-2}^{(i-1)}(f_o) \qquad (j = 1, 2, \ldots, 2i) \tag{40}$$

with the initial conditions:

$$B_j^{(i)}(f_o) = 0 \qquad (j \neq 0,\ \text{any}\ i)$$

$$B_0^{(i)}(f_o) = -1 \qquad (\text{Any}\ i)$$

The lone nonrecursive weight $A_o(f_o)$ for the cascaded filter for both the in-phase and quadrature filters is computed by

$$A_o(f_o) = \left| 1 - \sum_{j=1}^{V-1} B_j(f_o) \exp\left(-ij2\pi f_o'\right) \right| \tag{41}$$

where

V number of filter weights

$i = \sqrt{-1}$

The application of the band-pass filters is then

$$R_n^x(f_o) = \left[A_o^R(f_o)\right] x_n + \sum_{i=1}^{V-1} \left[B_i^R(f_o)\right]\left[R_{n-i}^x(f_o)\right] \qquad (n = 1, 2, \ldots, N) \tag{42}$$

$$I_n^x(f_o) = \left[A_o^I(f_o)\right] x_n + \sum_{i=1}^{V-1} \left[B_i^I(f_o)\right]\left[I_{n-i}^x(f_o)\right] \qquad (n = 1, 2, \ldots, N) \tag{43}$$

In order to minimize computer processing time and reduce disk storage, the program reduces the sampling rate of the data as much as possible for each filter output. These filter decimation factors are checked to insure that the new equivalent sampling rate is not a multiple of the filter's center frequency. Also, there must be, after reduction, a minimum of 50 points to analyze and 5 points per center frequency cycle. The filter decimation factors $D(f_o)$ are determined in the following manner:

Initially,

$$D(f_o) = I\left(\frac{1}{5d_p f_B'}\right) \tag{44}$$

where

d_p precondition decimation factor

f_B' normalized filter bandwidth associated with f_o

$I(x)$ integer part of x

Define

$$M(f_o) = I\left(2f_o' D(f_o)\, d_p\right) \tag{45}$$

where

f_o' normalized filter center frequency

If $M(f_o)$ is even,

$$\lambda_o = f'_o D(f_o)\, d_p - \frac{1}{2} M(f_o) \tag{46}$$

If $M(f_o)$ is odd,

$$\lambda_o = \frac{1}{2}\Big(M(f_o) + 1\Big) - f'_o D(f_o)\, d_p \tag{47}$$

If the following conditions are not satisfied, decrease $D(f_o)$ by 1, recompute $M(f_o)$ and λ_o, and try again:

$$\lambda_o + \frac{5}{4} f'_B \leqq 0.5 \tag{48}$$

$$\lambda_o - \frac{5}{4} f'_B \geqq 0 \tag{49}$$

$$\frac{N - N_W}{D(f_o)} \geqq 50 \tag{50}$$

where $N_W = I\left(\frac{1}{f'_B\, d_p}\right)$ = Number of filter warmup samples,

$$\frac{1}{f'_o\, D(f_o)} \geqq 5 \tag{51}$$

$$\left(f_s/f\right) \text{ modulo } D(f_o) \neq 0 \tag{52}$$

The conditions are checked in the order that they appear. If $D(f_o) = 1$ and either condition (48) or (49) fails, a diagnostic message is printed and the job is aborted. If $D(f_o) = 1$ and either condition (50), (51), or (52) fails, a diagnostic message is printed and the program continues to execute.

When the spectra are calculated by the band-pass filtering technique, there are no correlation computations performed, but additional information is obtained in the form of averages on the filter outputs (equivalent bandwidth and equivalent duration).

The equivalent bandwidth is a measure of the bandwidth of the filter as determined by the filter output. The bandwidth of each filter is computed as the root mean square of the instantaneous frequency around the average frequency. Small equivalent bandwidths (on the order of one-tenth of the filter bandwidth or less) may be the result of poor prewhitening, or the presence of a sinusoid.

The equivalent duration is a measure of the fraction of the time that the output of a filter is at high power. It is defined as the ratio of average power to peak power output of a filter. Thus, a sine wave would have an equivalent duration of 0.5, random noise would be expected to be in the neighborhood of 0.1, and transients or nonstationary data would be much smaller.

The equations for the spectral computations (noted only when different from the correlation technique) are as follows:

Power spectral density:

$$G_x(f_o) = \frac{\sum_{n=1}^{M} \left[\left(R_n^x(f_o) \right)^2 + \left(I_n^x(f_o) \right)^2 \right]}{2M \left| \beta(f_o) \right|} \tag{53}$$

where

M number of filter outputs after decimation

$\beta(f_o)$ noise bandwidth of the filter pair (computation method shown in appendix B)

Cross-spectral density:

$$C_{xy}(f_o) = \frac{\sum_{n=1}^{M} \left[R_n^x(f_o)\, R_n^y(f_o) + I_n^x(f_o)\, I_n^y(f_o) \right]}{M\beta(f_o)} \tag{54}$$

$$Q_{xy}(f_o) = \frac{\sum_{n=1}^{M} \left[R_n^x(f_o)\, I_n^y(f_o) - I_n^x(f_o)\, R_n^y(f_o) \right]}{-M\beta(f_o)} \tag{55}$$

In the equations for average frequency and equivalent bandwidth below, $\Delta\Phi_n^x(f_o)$ is defined by

$$\left.\begin{aligned} \Delta\Phi_n^x(f_o) &= \left[\tan^{-1}\left(\frac{-I_{n-1}^x(f_o)}{R_{n-1}^x(f_o)} \right) \right] - \left[\tan^{-1}\left(\frac{-I_n^x(f_o)}{R_n^x(f_o)} \right) \right] - 2\pi\lambda_o \qquad \left(M(f_o) \ \ \text{even} \right) \\ \Delta\Phi_n^x(f_o) &= \left[\tan^{-1}\left(\frac{-I_n^x(f_o)}{R_n^x(f_o)} \right) \right] - \left[\tan^{-1}\left(\frac{-I_{n-1}^x(f_o)}{R_{n-1}^x(f_o)} \right) \right] - 2\pi\lambda_o \qquad \left(M(f_o) \ \ \text{odd} \right) \end{aligned}\right\} \tag{56}$$

In both cases, $\Delta\Phi_n^x(f_o)$ is shifted to lie between $-\pi$ and π.

Average frequency:

$$\bar{f}_x(f_o) = f_o + \frac{\Delta\bar{\Phi}_x(f_o)}{\epsilon(f_o)} \tag{57}$$

where

$$\epsilon(f_o) = 2\pi\ \Delta t M D(f_o)\ d_p$$

$$\Delta\bar{\Phi}_x(f_o) = \frac{\sum_{n=1}^{M} \Delta\Phi_n^x(f_o)}{M}$$

Equivalent bandwidth:

$$B_x(f_o) = \sqrt{\frac{\Delta\Phi^2(f_o)}{4\pi^2(\Delta t)^2 D^2(f_o)\ d_p^2(M-1)} - \left(\bar{f}_x(f_o) - f_o\right)^2} \tag{58}$$

where

$$\Delta\Phi^2(f_o) = \frac{\sum_{n=1}^{M} \left(\Delta\Phi_n^x(f_o)\right)^2}{M}$$

Equivalent duration:

$$D_x(f_o) = \frac{\beta(f_o)\ G_x(f_o)}{\left[Z_n^x(f_o)\right]_{max}} \tag{59}$$

where

$$Z_n^x(f_o) = \left(R_n^x(f_o)\right)^2 + \left(I_n^x(f_o)\right)^2$$

Filter response function:

$$A_{fo}^a(f') = \left[\left(H_{fo}^a(f')\right)_{real}^2 + \left(H_{fo}^a(f')\right)_{imag}^2\right]^{1/2} \tag{60}$$

where the range of f' is given by the parameter cards and

$$H_{fo}^{a}(f') = \frac{A_o^a(f_o)}{1 - \sum_{n=1}^{y-1} B_n^a(f_o)\, e^{-2\pi f' n i}}$$

a R (in-phase) or I (quadrature)

$i = \sqrt{-1}$

$$\Phi_{fo}^{a}(f') = \tan^{-1} \frac{\left(H_{fo}^{a}(f')\right)_{imag}}{\left(H_{fo}^{a}(f')\right)_{real}}$$

$$\Delta\Phi_{fo}(f') = \Phi_{fo}^{R}(f') - \Phi_{fo}^{I}(f')$$

MODIFICATION OF SPECTRAL OUTPUTS

Several options are available for modification of the spectra outputs to reflect more accurately the spectra of the data of interest. These options may be chosen independently of each other and of the method chosen for obtaining the spectra. In each of the options where the power spectra and cross-spectra are modified, the functions which are determined by these spectra are computed by use of the modified spectra. The options are listed and the spectra modification equations are given in this section.

Amplitude and Phase Calibration Tables

A calibration of the instrumentation system amplitude response $H(f)$ and/or phase response $\beta(f)$ may be entered as a table of the variation of response with frequency for each channel. If the frequencies in the calibration tables do not match the frequency needed for the spectra, a linear interpolation is performed to obtain the proper response for the spectral frequency. This information is used to correct all spectral outputs for each channel in the following manner:

$$G_x(f) = \frac{G_x(f)}{H_x^2(f)} \tag{61}$$

$$C_{xy}(f) = \frac{C_{xy}(f)}{H_x(f)\, H_y(f)} \tag{62}$$

$$Q_{xy}(f) = \frac{Q_{xy}(f)}{H_x(f)\ H_y(f)} \tag{63}$$

$$\Phi_{xy}(f) = \Phi_{xy}(f) + \beta_x(f) - \beta_y(f) \tag{64}$$

Time Delay Calibration

A sinusoidal calibration signal may be applied simultaneously to all channels of the input data at time of data recording. If this signal is present, it may be used to compute a time lag τ_{xy} between each pair of channels x and y for which cross-spectra or cross-correlations are desired. The first 10 (or less if there are not 10) zero crossings are found by interpolation; they are then averaged and the difference between each set of two channels is taken as the time lag between those two channels. This time lag is then used to correct the phase of cross-spectral functions as follows:

$$\Phi_{xy}(f) = \Phi_{xy}(f) - 360f\tau_{xy} \tag{65}$$

Generally, the time delay calibration is not necessary unless the planned digitizing sampling frequency is very high and hence makes the possible small time difference between channels significant.

Shaping Filter Postdarkening

If the data were prewhitened with a shaping filter, the effect of this filter may then be removed from the spectra by using the transfer function of the filter $T(f)$. If the data were treated with a shaping filter to remove undesirable frequencies, the spectra should not be modified by the transfer function of the filter since this modification would simply insert the effect of these frequencies on the spectra, as if they were not removed. The filter transfer function is determined by

$$T^2(f) = \frac{P_N^2(f) + Q_N^2(f)}{P_D^2(f) + Q_D^2(f)} \tag{66}$$

where

$$P_N(f) = \sum_{j=0}^{N} A_j \cos(2\pi j f') \tag{67}$$

$$Q_N(f) = \sum_{j=0}^{N} A_j \sin(2\pi j f') \tag{68}$$

$$P_D(f) = 1 - \sum_{m=1}^{D} B_m \cos(2\pi m f') \tag{69}$$

$$Q_D(f) = \sum_{m=1}^{D} B_m \sin(2\pi m f') \tag{70}$$

If desired, the spectra are postdarkened as follows:

$$G_x(f) = \frac{G_x(f)}{T^2(f)} \tag{71}$$

$$C_{xy}(f) = \frac{C_{xy}(f)}{T^2(f)} \tag{72}$$

$$Q_{xy}(f) = \frac{Q_{xy}(f)}{T^2(f)} \tag{73}$$

PROBABILITY DENSITY ESTIMATION METHODS

Two techniques are available for the estimation of probability densities: (1) the standard histogram technique of dividing the amplitude of the data into intervals (bins) and counting the number of data points that fall in each bin, and (2) the smooth density function estimate obtained by an orthogonal series expansion.

The primary defects of the histogram method are:

(1) The division of the data into arbitrary bins may distort the distribution if there is curvature within a bin.

(2) A large number of bins are required to define the shape of a distribution and, as a result, a very jagged histogram is obtained.

The orthogonal function method does not have these defects but

(1) The distribution must be smooth to converge with 20 coefficients (orthogonal functions).

(2) The general shape of the distribution must be known.

(3) The approximation and "leakage" effects may cause some probability values to be negative.

(4) The chi-square normality test cannot be performed.

Unless the user is reasonably certain about the shape of the probability density function of his data, it is suggested that the histogram technique be used. This technique gives the user the shape of the probability density function, and the orthogonal function technique could then be used for obtaining a smooth probability distribution, if desired.

Histogram Technique

A histogram is computed for a channel of data by dividing the amplitude of the data into a specified number of bins. The number of data samples that lie in each bin are counted, and a plot is produced with data amplitude on the X-axis and counts on the Y-axis. If the parameter to subtract the mean was selected, then the mean was subtracted before the histogram was computed. Hence, to convert the histogram amplitude to actual data amplitude, one must add the mean for that channel. Provisions are made to combine adjacent bins in order to reach a specified number of counts per bin. This procedure tends to smooth the histogram at the low-count regions. Figure 8 gives an illustration of some digital input data and its corresponding histogram.

There are four options available for specifying the histogram parameters. For all four options, the number of bins is an input quantity. For option 1, the range of the histogram is determined by specifying the maximum and minimum histogram amplitude values; for option 2, the program scans the data for the maximum and minimum values and uses these values to determine the range of the histogram; for option 3, the maximum and minimum values are specified and bins are combined to reach an input-specified minimum number of counts in a bin; for option 4, the program scans the data for the maximum and minimum values and bins are combined as in option 3.

There is a restriction on the total number of bins for the input channels. If the correlation technique is used for spectral analysis, the total allowable number of bins is 1000. If the filtering technique is used for spectral analysis, the total number of bins allowed is reduced to 800.

Orthogonal Function Technique

By expanding the probability density into a finite orthonormal series and estimating the orthonormal coefficients from the data, a smooth estimate of the probability density function can be produced. The following two expansion choices are offered: Hermite functions which are appropriate for two-tailed distributions such as the Gaussian, chi-square for large degrees of freedom, and so forth; Laguerre functions which are appropriate for single-tailed distributions such as the exponential, gamma, or chi-square distribution for small degrees of freedom.

The equations for computing the orthonormal Hermite functions are as follows:

$$F_0(z_i) = \frac{\exp\left(-z_i^2/2\right)}{\pi^{1/4}} \tag{74}$$

$$F_1(z_i) = \sqrt{2}\, z_i F_o(z_i) \tag{75}$$

$$F_j(z_i) = \left(\frac{2}{j}\right)^{1/2} z_i F_{j-1}(z_i) - \left(\frac{j-1}{j}\right)^{1/2} F_{j-2}(z_i) \qquad (j = 2, 3, \ldots, 19) \tag{76}$$

where

$$z_i = \frac{x_i - \overline{X}}{\sigma}$$

The plots of these functions are shown in figure 9. The equations for computing the orthonormal Laguerre functions are as follows:

$$F_0(z_i) = \exp\left(\frac{-z_i}{2}\right) \tag{77}$$

$$F_1(z_i) = \left(1 - z_i\right) F_0(z_i) \tag{78}$$

$$F_j(z_i) = \frac{(2j - 1 - z_i)F_{j-1}(z_i) - (j - 1)F_{j-2}(z_i)}{j} \qquad (j = 2, 3, \ldots, 19) \tag{79}$$

where

$$z_i = \frac{2\left(x_i - b\right)}{\overline{X} - b}$$

$$b = \begin{cases} \text{minimum } x_i & \text{if tail goes toward maximum} \\ \text{maximum } x_i & \text{if tail goes toward minimum} \end{cases}$$

The plots of these functions are shown in figure 10. The coefficients for either expansion series are computed by

$$a_j = \frac{1}{N} \sum_{i=1}^{N} F_j(z_i) \tag{80}$$

where

N the number of data points

z_i normalized data point

The normalized probability density function $P(z)$ over the amplitude range specified in the input is then computed by a series expansion by using the appropriate series and the coefficients determined. The equation for this function is

$$P(z_i) = \sum_{j=1}^{P} a_j F_j(z_i) \tag{81}$$

where

P number of expansion terms (input parameter)

z_i normalized x_i

z determined by the range and increment specified on input cards

The probability density function $P(x)$ is then obtained from the normalized probability density function in the following manner:

For Hermite functions,

$$P(x) = \frac{P(z)}{\sigma} \tag{82}$$

For Laguerre functions,

$$P(x) = \frac{2P(z)}{|\overline{X} - b|} \tag{83}$$

Note that $P(x)$ and $P(z)$ are computed so that the area under each curve equals 1.

ADDITIONAL OUTPUTS

Many additional or subsidiary outputs are provided by the program for checking the form of the probability function, the digitizing process, off-scale values, randomness, and stationarity.

Average on the Entire Data

The mean, standard deviation, third (skewness), and fourth (kurtosis) normalized central moments of the data are computed by selecting the proper input parameter values. These values are always computed on the raw input data before preconditioning, if required, was performed. The equations for these values are as follows:

Mean:

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{84}$$

Standard deviation:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(x_i - \overline{X}\right)^2} \tag{85}$$

Skewness:

$$\beta = \frac{\frac{1}{N-1} \sum_{i=1}^{N} \left(x_i - \overline{X}\right)^3}{\sigma^3} \tag{86}$$

Kurtosis:

$$K = \frac{\frac{1}{N-1} \sum_{i=1}^{N} \left(x_i - \overline{X}\right)^4}{\sigma^4} \tag{87}$$

An interpretation that can be applied to the third and fourth normalized central moments is found in the discussion on "Distribution Tests."

Cross-Correlation Time Lag

If a time delay calibration (discussed earlier) is present, the true maximum time lag for cross-correlations τ^{ρ} may be computed. This value is based upon the time lag found between the two channels and the normal maximum time lag. Normally, the maximum time lag for cross-correlations is found by simply multiplying the maximum number of lags by the time increment. Therefore, the true maximum time lag between two channels is determined as follows:

$$\tau^{\rho}_{xy} = m\,\Delta t - \tau_{xy} \tag{88}$$

where

m maximum number of lags

τ_{xy} time lag between channels x and y

Distribution Tests

Several of the program's outputs may be used to test the data for the general shape of its distribution. For a two-tailed distribution, the shape is generally compared with the Gaussian distribution. For a single-tailed distribution, the shape is generally compared with the exponential distribution.

The third and fourth normalized central moments can give a rough idea of the distribution. To give the user some insight into the meaning of these moments, a Gaussian distribution has a skewness value of 0 and a kurtosis value of 3. A negative skewness value indicates the distribution has an elongated left tail and a positive skewness value indicates an elongated right tail. A distribution is called mesokurtic, platykurtic, or leptokurtic as its kurtosis equals 3, less than 3, or greater than 3, respectively. (See ref. 4.) A platykurtic distribution indicates the data points are less heavily concentrated about the mean and a leptokurtic distribution indicates the data points are more heavily concentrated, as compared with the Gaussian distribution.

The chi-square goodness of fit to a Gaussian distribution may be computed when the histogram option is chosen for the probability density function. The method of computing the chi-square value γ to test for normality is as follows:

$$\gamma = \sum_{i=1}^{K+2} \frac{\left(N_i - NP_i\right)^2}{NP_i} \tag{89}$$

where

P_i probability of being in ith bin if Gaussian

N_i number of points in ith bin

N total number of points

K degrees of freedom, number of bins $-$ 2

The region of normality acceptance is

$$\gamma \leqq \chi^2_{K,\alpha} \tag{90}$$

where the value of $\chi^2_{K,\alpha}$ is available from table VII and α is the probability of a type 1 error. As an example, suppose $\gamma = 65$, $K = 50$, and the data are to be tested for normality at the $\alpha = 0.05$ level of significance; then the data would be accepted to be normal since $\chi^2_{50,.05} = 67.5$ and $65 < 67.5$. (See ref. 5.)

When using the orthogonal function option for the probability density function, one can use the orthogonal coefficients to estimate the normality of the data. If the data are Gaussian, the zero-order coefficient in the Hermite expansion is three or four orders of magnitude greater than the remaining coefficients. If the data are exponentially distributed, the zero-order coefficient in the Laguerre expansion is three or four orders of magnitude greater than the remaining coefficients.

Accuracy Test

If the spectra were computed by the correlation technique, the program has an option for computing an accuracy measurement for the spectral estimators. For a specified confidence percentage, the program computes a percentage band for the spectral estimation. For example, a confidence percentage of 98 and a percentage band of 4.0 means that the user can be 98 percent certain that the spectral answers are within 4.0 percent of the true value. The equation for the percentage confidence band is as follows:

$$B_{pc} = \frac{S_{db}}{\sqrt{2\left(\frac{N}{m} - \frac{1}{3}\right) - 1}} \tag{91}$$

where

N	number of data points
m	maximum lag number
S_{db}	decibel spread for specified confidence percentage (see table VIII)

Off-Scale Test

The user can input by means of parameter cards an off-scale value and a maximum number of off-scale points for a channel. The program then checks each data point to see whether its absolute value exceeds the off-scale value. If it does, the data point, time, and replacement value is printed. When the maximum number of off-scale points for one channel is exceeded, the program aborts that case. The replacement value is the mean of all the preceding points. If the off-scale point is the first point, the replacement value equals zero.

The maximum and minimum value for each channel are determined after all off-scale values have been replaced and, if requested, the bias removed. These values are automatically printed to give the user the range of the data analyzed.

CONCLUDING REMARKS

This paper was prepared as a reference guide to users of the Langley time series analysis computer program (designated R1299), which is a general purpose program for the analysis of random, stationary time series. The bases for choosing available options, descriptions of the options, possible output quantities, equations of the various computations, and tables and figures which might aid in the analysis of data were included.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., December 1, 1970.

APPENDIX A

GENERAL COMPUTER PROCEDURE

The program is set up so that any of the 20 channels may be processed according to parameter card options. Cross-correlations or cross-spectra may be computed between all possible combinations of any two channels.

The following procedure is used:

(a) All parameters and calibration data are read and the time lag, if any, is computed.

(b) The data file is read and the mean, if requested, is computed for each channel. The data are placed on a blocked disk file.

(c) When phase II (filtering method) is used, the filter weights are computed and the filter response, if requested, is computed, printed, and placed on a binary output tape.

(d) A data point is read from the data disk file.

(e) Probability density function estimators, if requested, are updated for all channels.

(f) The statistics (standard deviation, third central moment, and so forth) summations for all channels are updated.

(g) The data point is conditioned according to parameter card options.

(h) This step is the first one in which all the channels may not be processed together. The number of channels processed in one data pass is a function of the number of lags in the correlation method and of the number of weights and filters in the filtering method.

For correlation method:

(1) The autocovariance and cross-covariance sums for channels to process during this data pass are updated.

(2) If all the data have not been read, return to step (d).

For filtering method:

(1) The channels to process during this data pass are filtered and placed on a scratch disk file.

(2) If all the data have not been read, return to step (d).

(i) After all the data have been read, the probability density function estimations, if requested, are completed, printed, and placed on the binary output tape. The statistics are completed and printed.

(j) For correlation method:

(1) The autocovariance and cross-covariance functions are completed, printed, and placed on the binary output tape.

(2) The autocorrelation, cross-correlation, spectra, etc., are computed, printed, and placed on the binary output tape.

(3) If there are more channels to process (that is, another data pass is necessary), rewind the data disk file and return to step (d) and eliminate steps (e), (f), and (i).

For filtering method:

(1) If there are more channels to process (that is, another data pass is necessary), rewind the data disk file and return to step (d) and eliminate steps (e), (f), and (i).

(k) Filtering method only:

(1) The spectral summations are computed from the filtered data on the proper scratch disk file.

(2) The final spectra, equivalent bandwidth, and so forth answers are computed, printed, and placed on the binary output file.

(3) Return to step k(1) if more than one data pass was required to filter all the channels.

(l) After all the channels have been processed, the plot subroutines are entered and the plot instructions are generated by use of the binary output file. If another data case is present, the program will return to step (a); if not, it will terminate.

For the correlation method, the number of channels processed in one data pass in steps (h) and (j) depends upon the maximum number of lags desired. There are 2000 storage locations allotted for single-channel processing and 2000 storage locations allotted for cross-channel processing. Tables III and IV show the maximum number of single channels and cross channels that can be performed in one data pass for a given number of lags.

For the filtering method, the number of channels processed in one data pass in steps (h), (j), and (k) depends upon the number of filter weights and the number of filters that must be generated for the frequency analysis. There are 2000 storage locations allotted for the filter weights and 2000 storage locations allotted for storing the previous filter outputs from each channel. Tables V and VI show the maximum number of single channels or cross channels that can be performed in one data pass for a given number of filter weights and filters that must be generated. When possible, an even number of channels are processed in one data pass to permit the maximum number of cross-channel calculations with the minimum amount of time and filtering.

APPENDIX B

NOISE BANDWIDTH COMPUTATIONAL METHOD FOR FILTER PAIR

The equation for computing the noise bandwidth for the analysis pair of filters used in the filtering technique is as follows:

$$\beta(f_o) = \frac{1}{\Delta t\ dp} \int_0^{1/2} H(f_o,\alpha)\, d\alpha$$

$$= \frac{1}{\Delta t\ dp} \left[\int_0^{\alpha_o(f_o)-\Delta\alpha(f_o)} H(f_o,\alpha)\, d\alpha + \int_{\alpha_o(f_o)-\Delta\alpha(f_o)}^{\alpha_o(f_o)+\Delta\alpha(f_o)} H(f_o,\alpha)\, d\alpha + \int_{\alpha_o(f_o)+\Delta\alpha(f_o)}^{1/2} H(f_o,\alpha)\, d\alpha \right]$$

where

$$\alpha_o(f_o) = f_o\ \Delta t\ dp$$

$$\Delta\alpha(f_o) = \begin{cases} \alpha_o(B_1) & (f_o = f_1) \\ \alpha_o(f_o) - \alpha_o(f_o^*) & (f_o = f_2,\ f_3,\ \ldots,\ f_{max}) \end{cases}$$

B_1	filter bandwidth associated with f_1
f_1	center frequency of first filter
f_2	center frequency of second filter
f_o^*	center frequency of filter preceding f_o

$$H(f_o,\alpha) = \frac{P(f_o,\alpha) + Q(f_o,\alpha)}{2}$$

$$P(f_o,\alpha) = \frac{\left[A_o^R(f_o)\right]^2}{\left(1 - \sum_{i=1}^{V-1} B_i^R(f_o)\cos(2\pi\alpha i)\right)^2 + \left(\sum_{i=1}^{V-1} B_i^R(f_o)\sin(2\pi\alpha i)\right)^2}$$

$$Q(f_o,\alpha) = \frac{\left[A_o^I(f_o)\right]^2}{\left(1 - \sum_{i=1}^{V-1} B_i^I(f_o)\cos(2\pi\alpha i)\right)^2 + \left(\sum_{i=1}^{V-1} B_i^I(f_o)\sin(2\pi\alpha i)\right)^2}$$

Simpson's rule for integration with 90 intervals is used in each of the 3 regions to evaluate the integrals and obtain the final result.

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TABLE I.- DYNAMIC DATA RECORD FORMAT FOR TSA PROGRAM

Word number	Mode[a]	Description
1	Floating	Number of channels in the record
2	Floating	Serial number
3	Fixed (Hollerith)[b]	Primary engineering identification
4	Fixed (Hollerith)[b]	Primary engineering identification
5	Fixed (Hollerith)[b]	Additional engineering identification
6	Fixed (Hollerith)[b]	Additional engineering identification
7	Fixed (Hollerith)[b]	Additional engineering identification
8	Fixed (Hollerith)[b]	Additional engineering identification
9	Floating	Record count
10	Floating	Elapsed time in seconds
11	Floating	Channel 1 data sample
12	Floating	Channel 2 data sample
.	.	.
.	.	.
.	.	.
N + 10	Floating	Channel N data sample where N is the number in word 1

[a]Data should be written on tape as a binary file using a floating-point array.

[b]Each word contains 10 characters.

TABLE II.- BLOCKED FORMAT FOR TSA PROGRAM

Number of channels, N	Words per record	Records per block[a]	Words per block[a]
$1 \leqq N \leqq 10$	20	25	500
$10 < N \leqq 20$	30	17	510

[a]A block is a FORTRAN logical record.

TABLE III.- MAXIMUM NUMBER OF CROSSES PROCESSED IN ONE PASS FOR CORRELATION METHOD

Number of lags desired	Maximum number of crosses processed in one data pass
500 to 999	1
334 to 499	2
250 to 333	3
200 to 249	4
167 to 199	5
143 to 166	6
125 to 142	7
112 to 124	8
100 to 112	9
91 to 99	10

TABLE IV.- MAXIMUM NUMBER OF SINGLE CHANNELS PROCESSED IN ONE PASS FOR CORRELATION METHOD

Number of lags desired	Maximum number of single channels processed in one data pass
667 to 999	2
500 to 666	3
400 to 499	4
334 to 399	5
286 to 333	6
250 to 285	7
223 to 249	8
200 to 222	9
182 to 199	10
167 to 181	11
154 to 166	12
143 to 153	13
134 to 142	14
125 to 133	15
118 to 124	16
112 to 117	17
106 to 111	18
100 to 105	19
0 to 99	20

TABLE V.- MAXIMUM NUMBER OF CROSSES PROCESSED IN ONE PASS FOR FILTERING METHOD

Number of filter weights	Number of filters generated	Maximum number of crosses processed in one data pass
5	201 to 400	0
5	101 to 200	1
5	67 to 100	2
5	51 to 66	3
5	41 to 50	4
7	143 to 285	0
7	72 to 142	1
7	48 to 71	2
7	36 to 47	3
7	29 to 35	4
9	112 to 222	0
9	56 to 111	1
9	38 to 55	2
9	28 to 37	3
9	23 to 27	4

TABLE VI.- MAXIMUM NUMBER OF SINGLE CHANNELS PROCESSED IN ONE PASS FOR FILTERING METHOD

Number of filtering weights	Number of filters generated	Maximum number of single channels processed in one data pass
5	201 to 400	1
5	101 to 200	2
5	67 to 100	4
5	51 to 66	6
5	41 to 50	8
7	143 to 285	1
7	72 to 142	2
7	48 to 71	4
7	36 to 47	6
7	29 to 35	8
9	112 to 222	1
9	56 to 111	2
9	38 to 55	4
9	28 to 37	6
9	23 to 27	8

TABLE VII.- CHI-SQUARE GOODNESS-OF-FIT TEST

K	$\alpha = 0.20$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	$\alpha = 0.005$	$\alpha = 0.001$	$\alpha = 0.0005$
1	1.64	2.03	2.71	3.84	5.41	6.64	7.88	11.2	12.6
2	3.22	3.76	4.60	5.99	7.82	9.21	10.60	14.1	15.7
3	4.64	5.29	6.25	7.82	9.84	11.3	12.80	16.6	18.1
4	5.99	6.72	7.78	9.49	11.7	13.3	14.9	18.7	20.4
5	7.29	8.09	9.24	11.1	13.4	15.1	16.8	20.8	22.4
6	8.56	9.42	10.6	12.6	15.0	16.8	18.6	22.7	24.4
7	9.80	10.7	12.0	14.0	16.6	18.5	20.3	24.5	26.3
8	11.0	12.0	13.4	15.5	18.2	20.1	22.0	26.3	28.1
9	12.2	13.3	14.7	16.9	19.7	21.7	23.6	28.1	29.9
10	13.4	14.5	16.0	18.3	21.2	23.2	25.2	29.8	31.7
11	14.6	15.8	17.3	19.7	22.6	24.8	26.8	31.4	33.4
12	15.8	16.9	18.5	21.0	24.1	26.3	28.3	33.1	35.0
13	17.0	18.2	19.8	22.4	25.5	27.7	29.8	34.7	36.7
14	18.2	19.4	21.0	23.7	26.9	29.1	31.3	36.3	38.3
15	19.3	20.6	22.3	25.0	28.3	30.6	32.8	37.8	39.9
16	20.5	21.8	23.5	26.3	29.6	32.0	34.3	39.4	41.5
17	21.6	23.0	24.8	27.6	31.0	33.4	35.7	40.9	43.1
18	22.8	24.1	26.0	28.9	32.3	34.8	37.2	42.5	44.6
19	23.9	25.3	27.2	30.1	33.7	36.2	38.6	44.0	46.2
20	25.0	26.5	28.4	31.4	35.0	37.6	40.0	45.4	47.7
21	26.2	27.7	29.6	32.7	36.3	38.9	41.4	46.9	49.2
22	27.3	28.8	30.8	33.9	37.7	40.3	42.8	48.4	50.7
23	28.4	30.0	32.0	35.2	39.0	41.7	44.2	49.9	52.2
24	29.6	31.1	33.2	36.4	40.3	43.0	45.6	51.3	53.7
25	30.7	32.3	34.4	37.6	41.6	44.3	46.9	52.7	55.1
26	31.8	33.4	35.6	38.9	42.9	45.6	48.3	54.2	56.6
27	32.9	34.6	36.7	40.1	44.1	47.0	49.6	55.6	58.0
28	34.0	35.7	37.9	41.3	45.4	48.3	51.0	57.0	59.4
29	35.1	36.8	39.1	42.6	46.7	49.6	52.3	58.4	60.9
30	36.3	38.0	40.3	43.8	48.0	50.9	53.7	59.8	62.3
31	37.4	39.1	41.4	45.0	49.2	52.2	55.1	61.2	63.7
32	38.5	40.3	42.6	46.2	50.5	53.5	56.4	62.6	65.1
33	39.6	41.4	43.7	47.4	51.8	54.8	57.7	64.0	66.5
34	40.7	42.5	44.9	48.6	53.0	56.1	59.0	65.4	67.9
35	41.8	43.6	46.1	49.8	54.3	57.4	60.3	66.7	69.3
36	42.9	44.8	47.2	51.0	55.5	58.6	61.6	68.1	70.7
37	44.0	45.9	48.4	52.2	56.7	59.9	62.9	69.4	72.1
38	45.1	47.0	49.5	53.4	58.0	61.2	64.2	70.8	73.5
39	46.2	48.1	50.7	54.6	59.2	62.5	65.5	72.2	74.9
40	47.3	49.2	51.8	55.8	60.4	63.7	66.8	73.5	76.2

TABLE VII.- CHI-SQUARE GOODNESS-OF-FIT TEST – Continued

K	α = 0.20	α = 0.15	α = 0.10	α = 0.05	α = 0.02	α = 0.01	α = 0.005	α = 0.001	α = 0.0005
41	48.4	50.4	52.9	56.9	61.7	65.0	68.1	74.8	77.6
42	49.5	51.5	54.1	58.1	62.9	66.2	69.4	76.2	78.9
43	50.5	52.6	55.2	59.3	64.1	67.5	70.7	77.5	80.3
44	51.6	53.7	56.4	60.5	65.3	68.7	71.9	78.8	81.6
45	52.7	54.8	57.5	61.7	66.6	70.0	73.2	80.2	83.0
46	53.8	55.9	58.6	62.8	67.8	71.2	74.5	81.5	84.3
47	54.9	57.0	59.8	64.0	69.0	72.5	75.7	82.8	85.7
48	56.0	58.1	60.9	65.2	70.2	73.7	77.0	84.1	87.0
49	57.1	59.2	62.0	66.3	71.4	74.9	78.3	85.4	88.3
50	58.2	60.3	63.2	67.5	72.6	76.2	79.5	86.7	89.7
51	59.2	61.4	64.3	68.7	73.8	77.4	80.8	88.1	91.0
52	60.3	62.5	65.4	69.8	75.0	78.6	82.0	89.4	92.3
53	61.4	63.7	66.5	71.0	76.2	79.9	83.3	90.6	93.6
54	62.5	64.8	67.7	72.2	77.4	81.1	84.5	92.0	94.9
55	63.6	65.9	68.8	73.3	78.6	82.3	85.8	93.3	96.3
56	64.7	66.9	69.9	74.5	79.8	83.5	87.0	94.5	97.6
57	65.7	68.0	71.0	75.6	81.0	84.8	88.3	95.8	98.9
58	66.8	69.1	72.2	76.8	82.2	86.0	89.5	97.1	100.2
59	67.9	70.2	73.3	78.0	83.4	87.2	90.8	98.4	101.5
60	69.0	71.3	74.4	79.1	84.6	88.4	92.0	99.7	102.8
61	70.0	72.4	75.5	80.2	85.8	89.6	93.2	101.0	104.1
62	71.1	73.5	76.6	81.4	87.0	90.8	94.5	102.2	105.4
63	72.2	74.6	77.7	82.5	88.1	92.0	95.7	103.5	106.7
64	73.3	75.7	78.9	83.7	89.3	93.2	96.9	104.8	108.0
65	74.3	76.8	80.0	84.8	90.5	94.4	98.1	106.1	109.3
66	75.4	77.9	81.1	86.0	92.7	95.6	99.4	107.3	110.5
67	76.5	79.0	82.2	87.1	92.9	96.9	100.6	108.6	111.8
68	77.6	80.1	83.3	88.3	94.0	98.1	101.8	109.9	113.2
69	78.6	81.2	84.4	89.4	95.2	99.3	103.0	111.2	114.4
70	79.7	82.3	85.5	90.6	96.4	100.4	104.3	112.4	115.7
71	80.8	83.3	86.6	91.7	97.6	101.6	105.5	113.7	116.9
72	81.9	84.4	87.7	92.8	98.7	102.8	106.7	114.9	118.2
73	82.9	85.5	88.8	93.9	99.9	104.0	107.9	116.2	119.5
74	84.0	86.6	90.0	95.1	101.1	105.2	109.1	117.4	120.8
75	85.1	87.7	91.1	96.2	102.3	106.4	110.3	118.7	122.0
76	86.1	88.8	92.2	97.4	103.4	107.6	111.5	119.9	123.3
77	87.2	89.9	93.3	98.5	104.6	108.8	112.7	121.2	124.6
78	88.3	90.9	94.4	99.6	105.8	110.0	113.9	122.4	125.8
79	89.3	92.0	95.5	100.8	106.9	111.2	115.2	123.7	127.1
80	90.4	93.1	96.6	101.9	108.1	112.4	116.4	124.9	128.4

TABLE VII.- CHI-SQUARE GOODNESS-OF-FIT TEST – Concluded

K	α = 0.20	α = 0.15	α = 0.10	α = 0.05	α = 0.02	α = 0.01	α = 0.005	α = 0.001	α = 0.0005
81	91.5	94.2	97.7	103.0	109.2	113.5	117.6	126.2	129.6
82	92.5	95.3	98.8	104.1	110.4	114.7	118.8	127.4	130.9
83	93.6	96.3	99.9	105.3	111.6	115.9	120.0	128.6	132.1
84	94.7	97.4	101.0	106.4	112.7	117.1	121.2	129.9	133.4
85	95.7	98.5	102.1	107.5	113.9	118.3	122.4	131.1	134.6
86	96.8	99.6	103.2	108.7	115.0	119.4	123.6	132.3	135.9
87	97.9	100.7	104.3	109.8	116.2	120.6	124.8	133.6	137.1
88	98.9	101.7	105.4	110.9	117.4	121.8	125.9	134.8	138.4
89	100.0	102.8	106.5	112.0	118.5	123.0	127.1	136.0	139.6
90	101.1	103.9	107.6	113.1	119.7	124.1	128.3	137.3	140.9
91	102.1	105.0	108.7	114.3	120.8	125.3	129.5	138.5	142.1
92	103.2	106.1	109.8	115.4	122.0	126.5	130.7	139.7	143.4
93	104.2	107.1	110.8	116.5	123.1	127.7	131.9	141.0	144.6
94	105.3	108.2	111.9	117.6	124.3	128.8	133.1	142.2	145.8
95	106.4	109.3	113.0	118.8	125.4	130.0	134.3	143.4	147.1
96	107.4	110.4	114.1	119.9	126.6	131.2	135.5	144.6	148.3
97	108.5	111.4	115.2	121.0	127.7	132.3	136.7	145.9	149.5
98	109.5	112.5	116.3	122.1	128.9	133.5	137.8	147.1	150.8
99	110.6	113.6	117.4	123.2	130.0	134.7	139.0	148.3	152.0
100	111.7	114.7	118.5	124.3	131.2	135.8	140.2	149.5	153.2
105	116.9	120.0	123.9	129.9	136.9	141.6	146.1	155.6	159.4
110	122.2	125.4	129.4	135.5	142.6	147.4	152.0	161.6	165.5
115	127.5	130.7	134.8	141.0	148.3	153.2	157.8	167.7	171.6
120	132.8	136.1	140.2	146.6	153.9	159.0	163.7	173.7	177.7
125	138.1	141.4	145.6	152.1	159.6	164.7	169.5	179.7	183.7
130	143.3	146.7	151.0	157.6	165.2	170.4	175.3	185.6	189.8
135	148.6	152.0	156.4	163.1	170.9	176.2	181.1	191.6	195.8
140	153.9	157.4	161.8	168.6	176.5	181.9	186.9	197.5	201.8
145	159.1	162.7	167.2	174.1	182.1	187.6	192.6	203.4	207.7
150	164.3	167.9	172.6	179.6	187.7	193.2	198.4	209.3	213.7
160	174.8	178.6	183.3	190.5	198.9	204.6	209.9	221.1	225.5
170	185.3	189.1	194.0	201.4	210.0	215.8	221.3	232.8	237.4
180	195.7	199.7	204.7	212.3	221.1	227.1	232.7	244.4	249.1
190	206.2	210.2	215.4	223.2	232.2	238.3	244.0	256.0	260.8
200	216.6	220.7	226.0	234.0	243.2	249.5	255.3	267.6	272.5
220	237.4	241.8	247.3	255.6	265.2	271.7	277.8	290.6	295.7
240	258.2	262.7	268.5	277.2	287.1	293.9	300.2	313.5	318.8
260	279.0	283.7	289.6	298.6	308.9	316.0	322.5	336.2	341.7
280	299.7	304.5	310.7	320.0	330.7	338.0	344.7	358.9	364.5
300	320.4	325.4	331.8	341.4	352.4	359.9	366.9	381.5	387.3

TABLE VIII.- DECIBEL SPREAD FOR SPECIFIED CONFIDENCE PERCENTAGES

Decibel spread	Specified confidence percentages
6	40
7.8	50
10	60
16	80
20	90
24.2	95
25	96
29	98
32	99

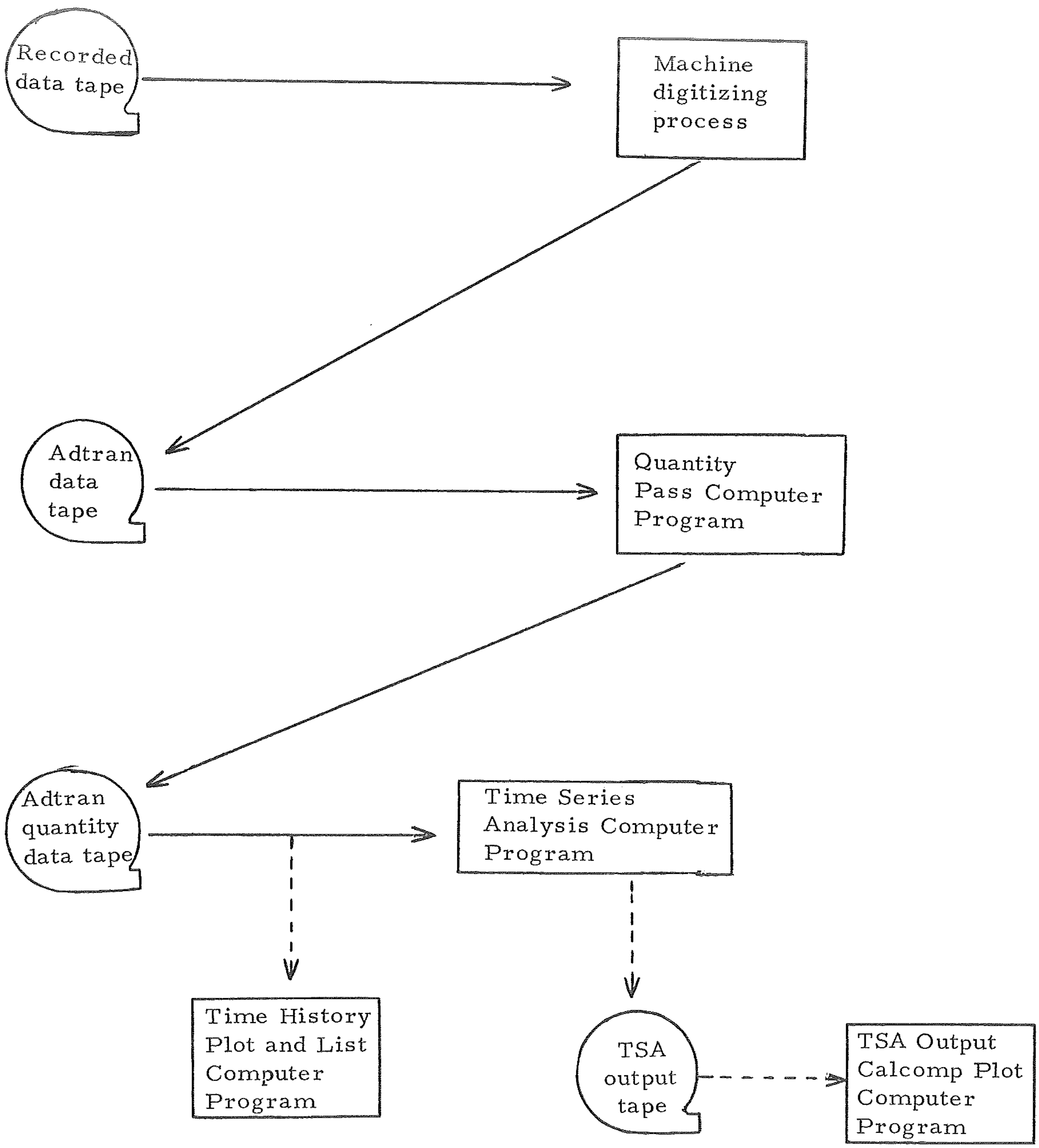


Figure 1.- Dynamic data reduction process.

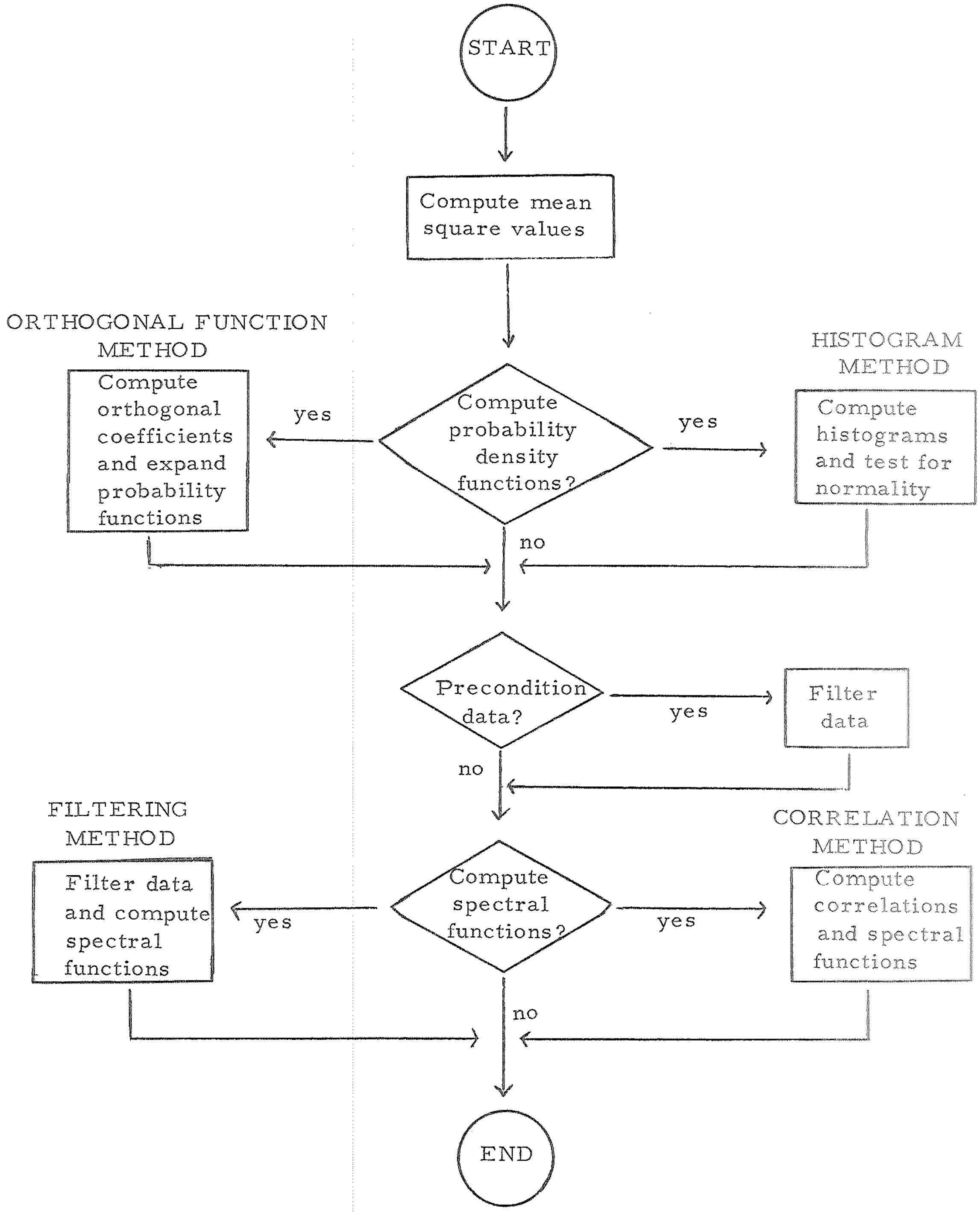
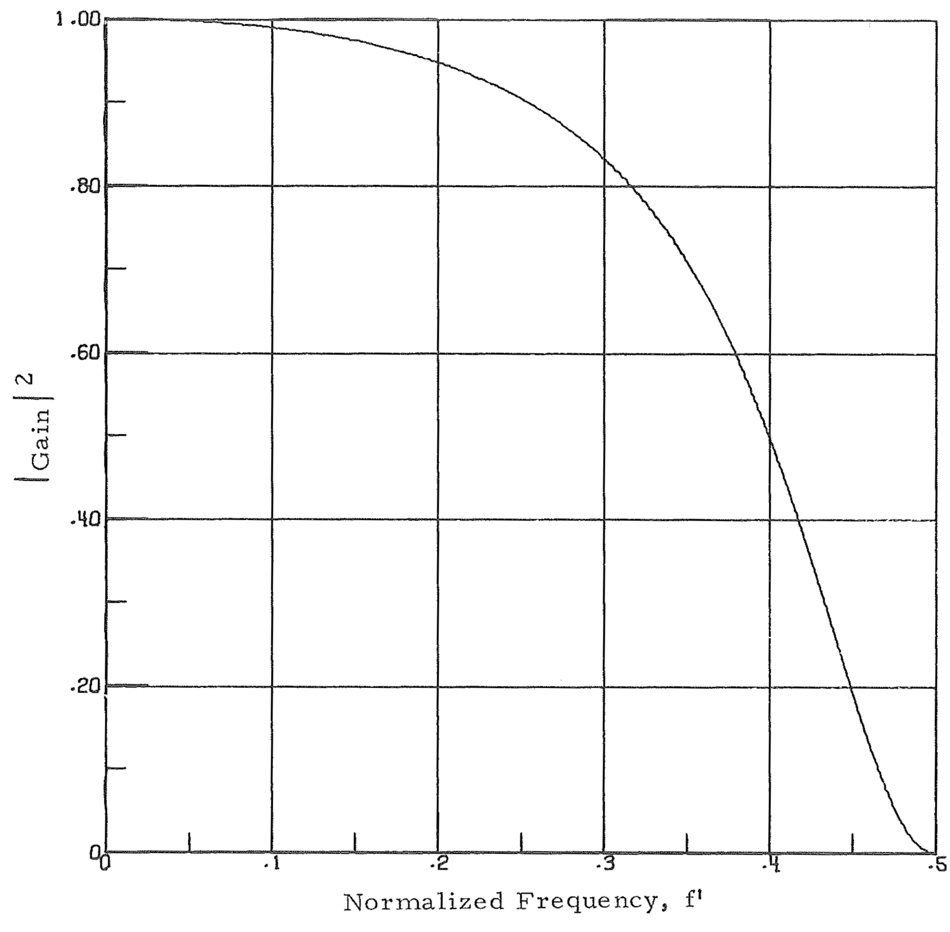
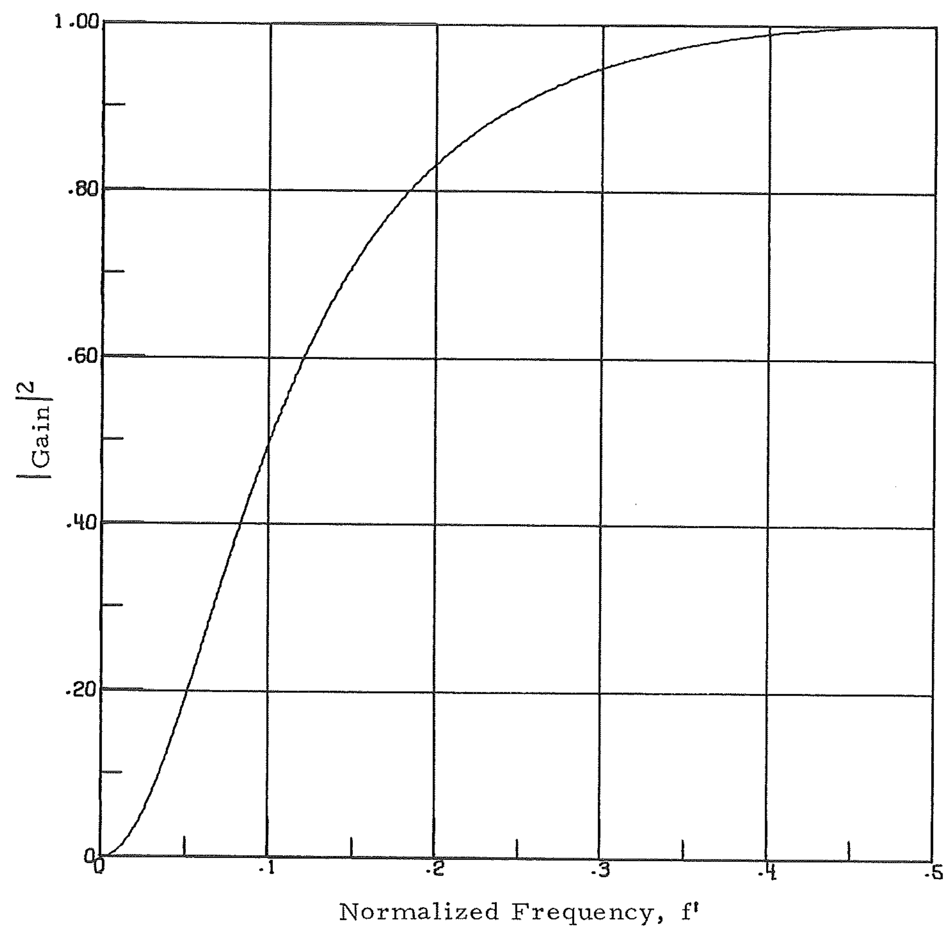


Figure 2.- Simplified flow through TSA program.



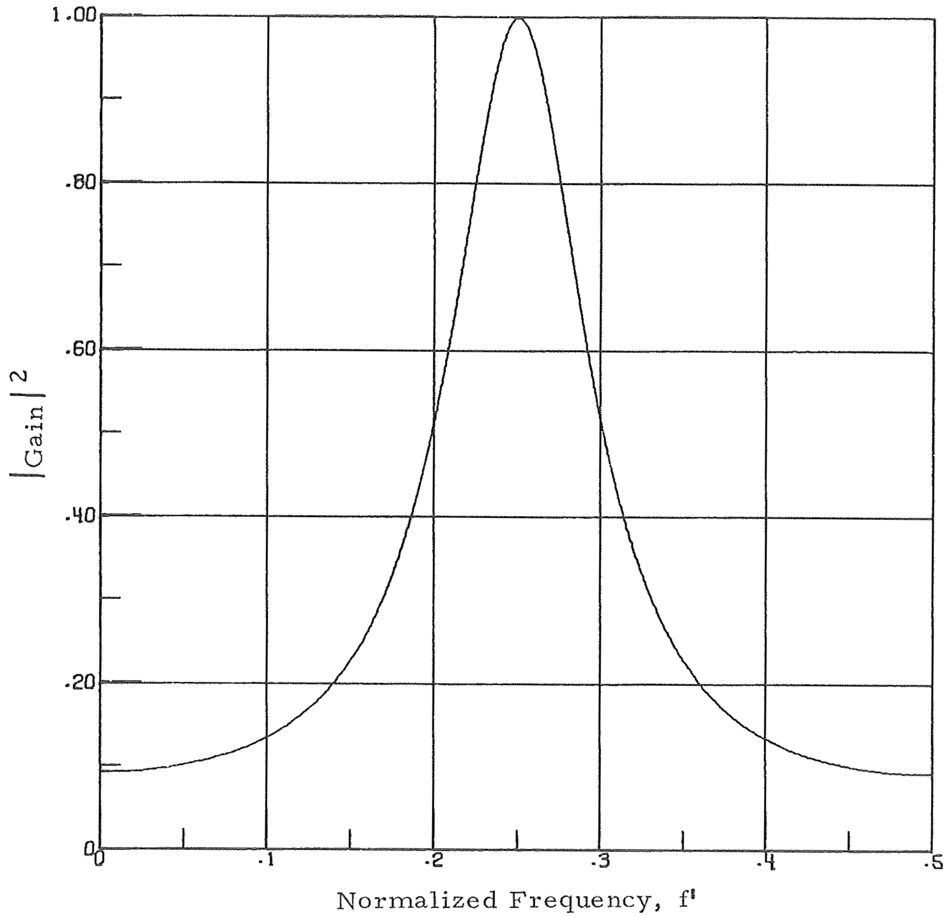
(a) Low pass.

Figure 3.- Precondition simple filters.



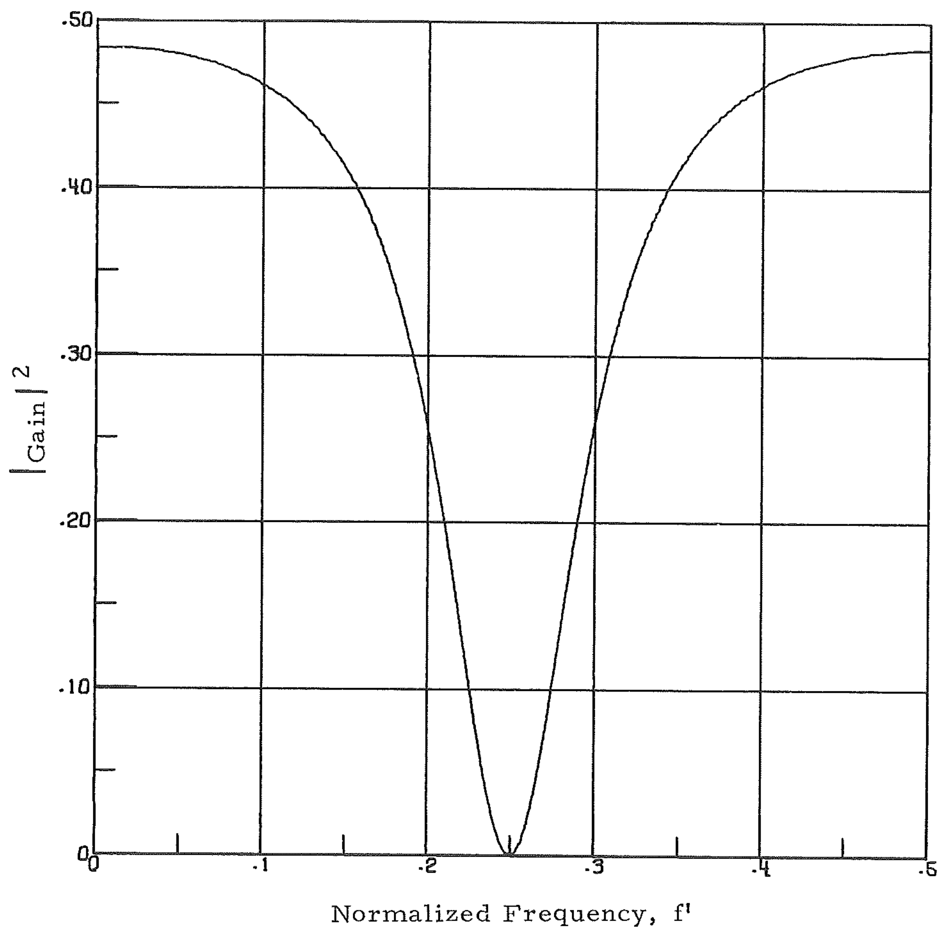
(b) High pass.

Figure 3.- Continued.



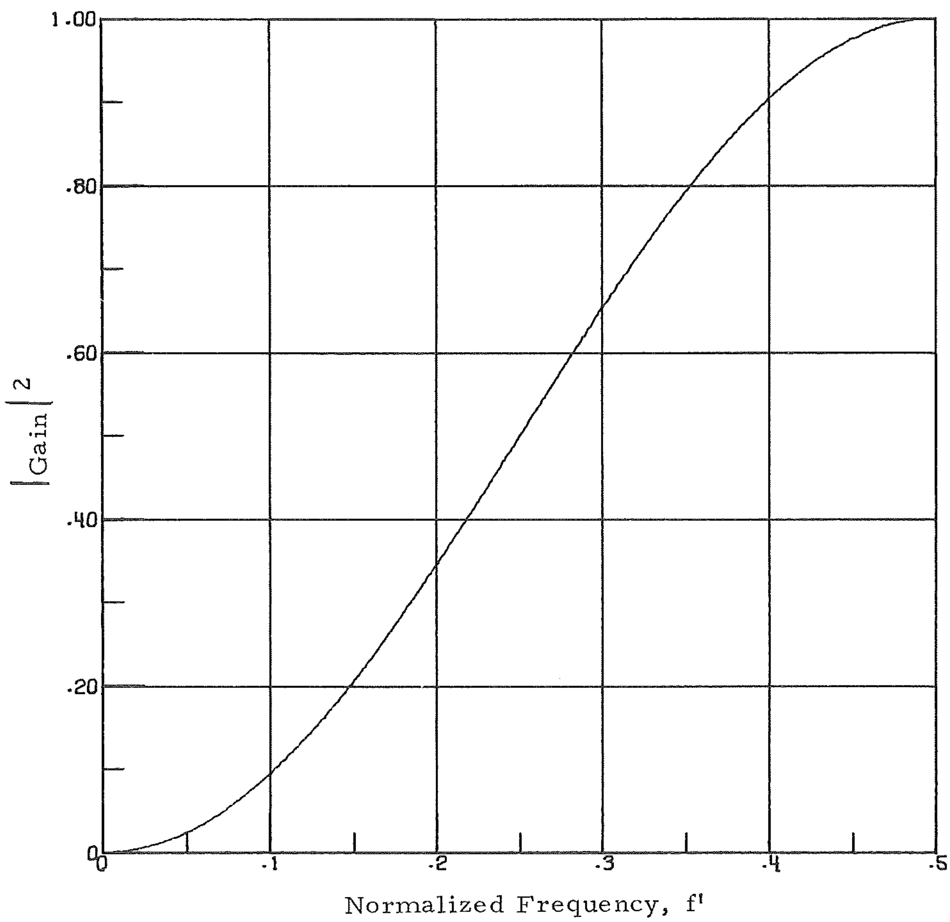
(c) Band pass

Figure 3.- Continued.



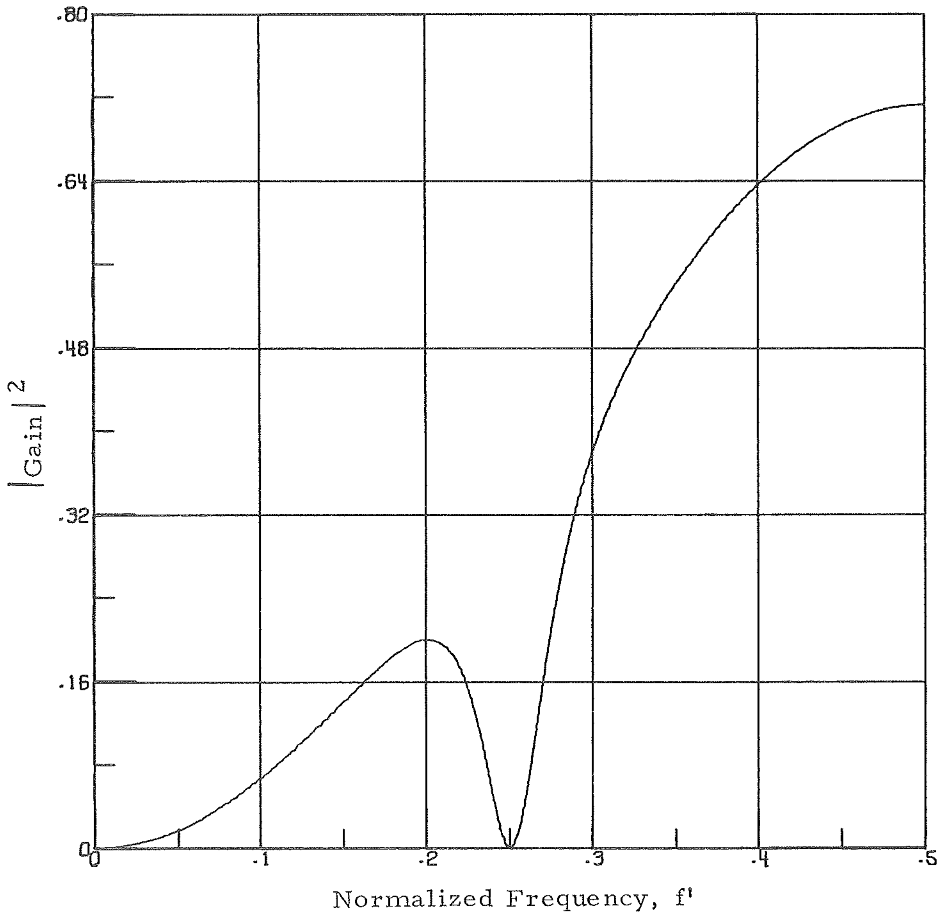
(d) Band reject.

Figure 3.- Continued.



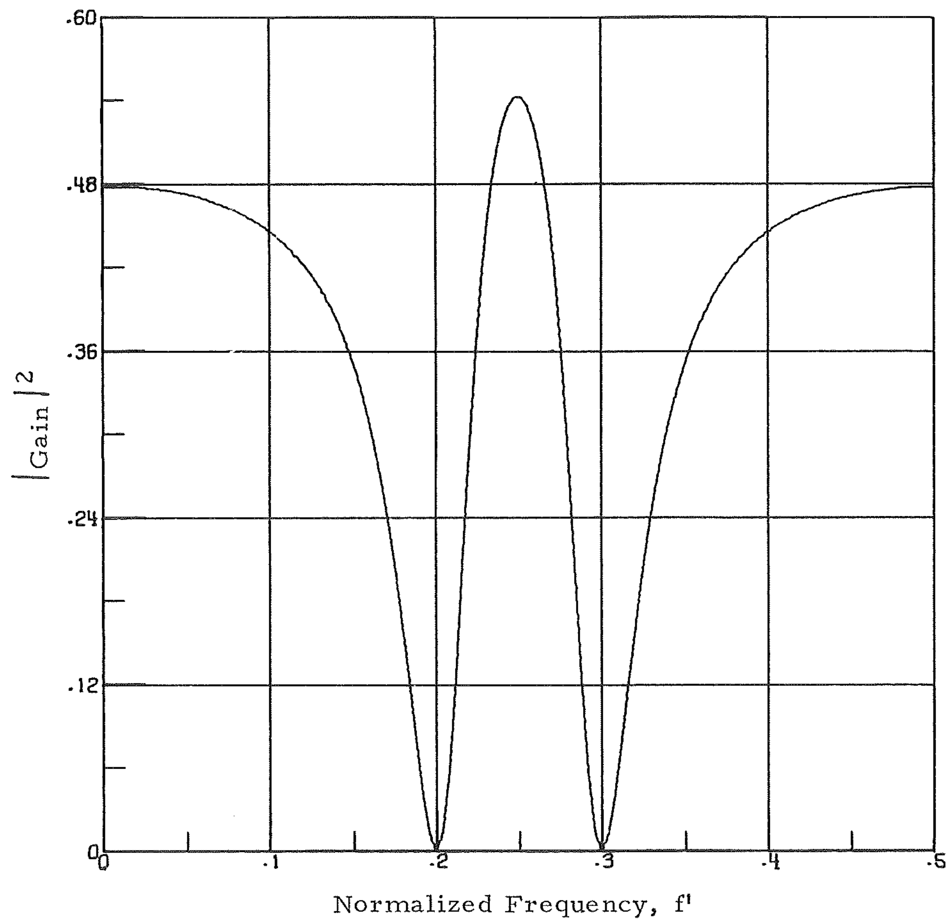
(e) First difference.

Figure 3.- Concluded.



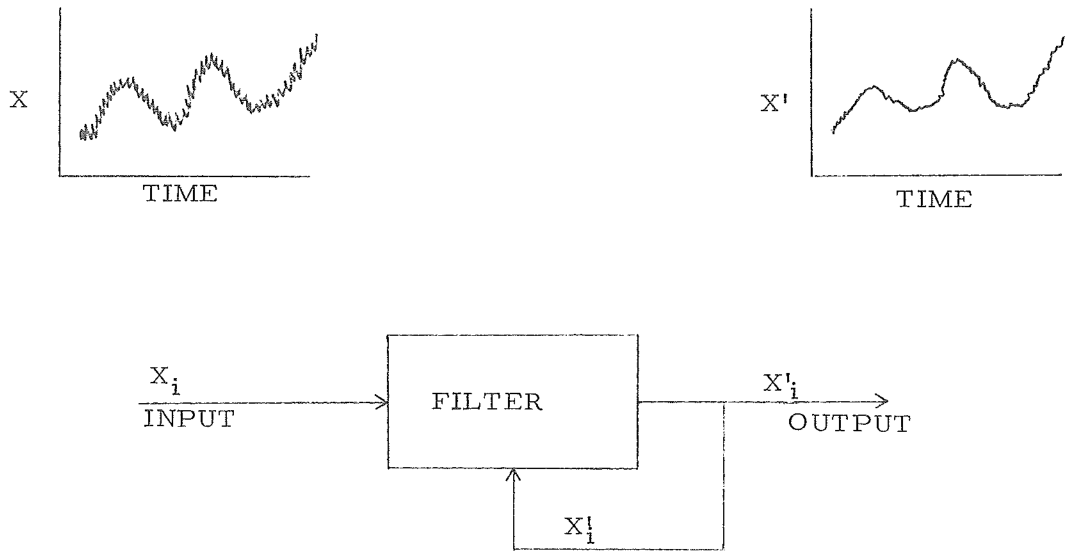
(a) First difference and band reject cascade.

Figure 4.- Examples of precondition compound filters.



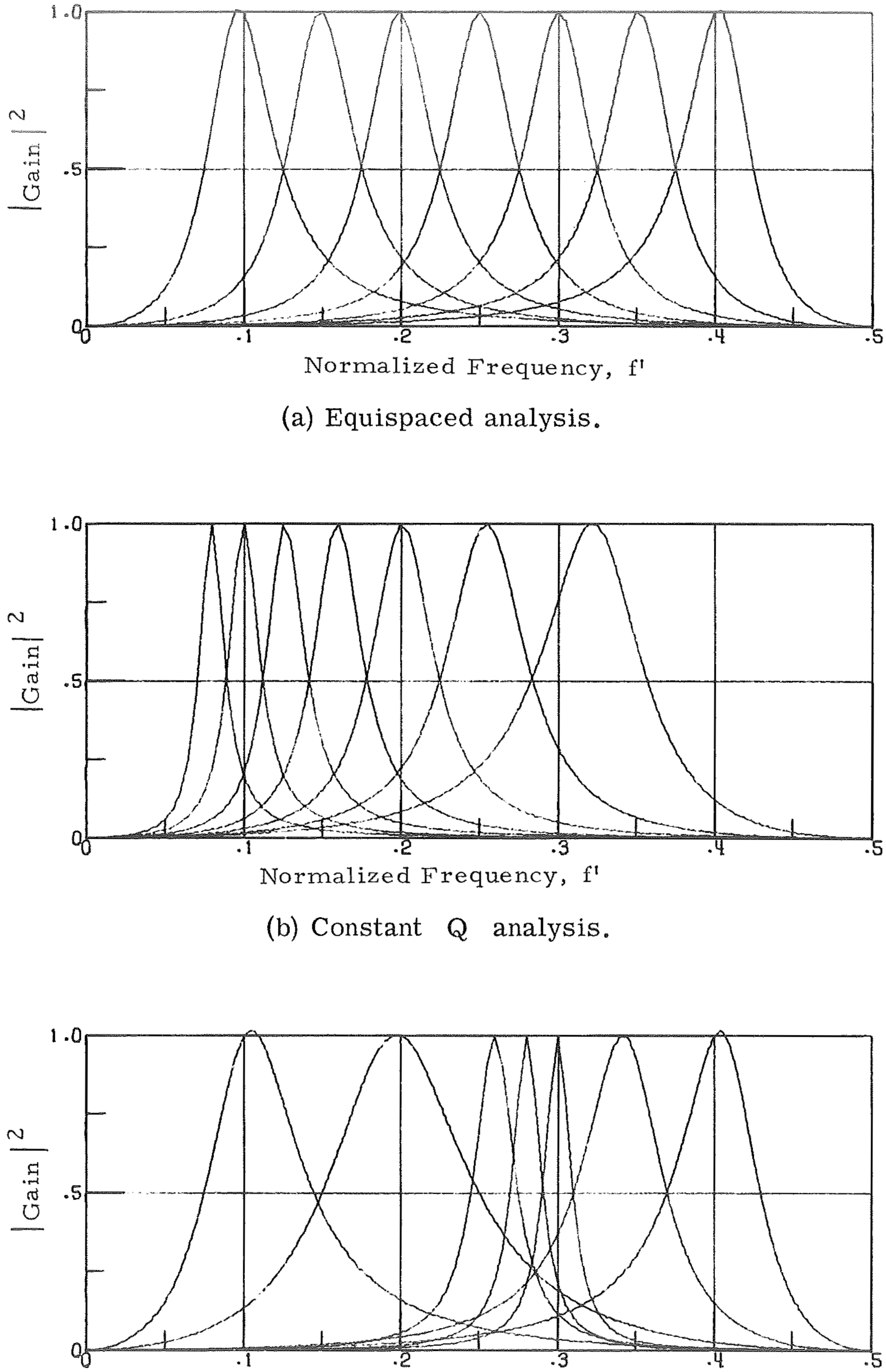
(b) Band reject and band reject cascade.

Figure 4.- Concluded.



$$X'_i = \sum_{j=0}^{N} A_j X_{i-j} + \sum_{m=1}^{D} B_m X'_{i-m}$$

Figure 5.- Recursive filter.



(a) Equispaced analysis.

(b) Constant Q analysis.

(c) Arbitrary analysis.

Figure 6.- Examples of the analysis band-pass filter options.

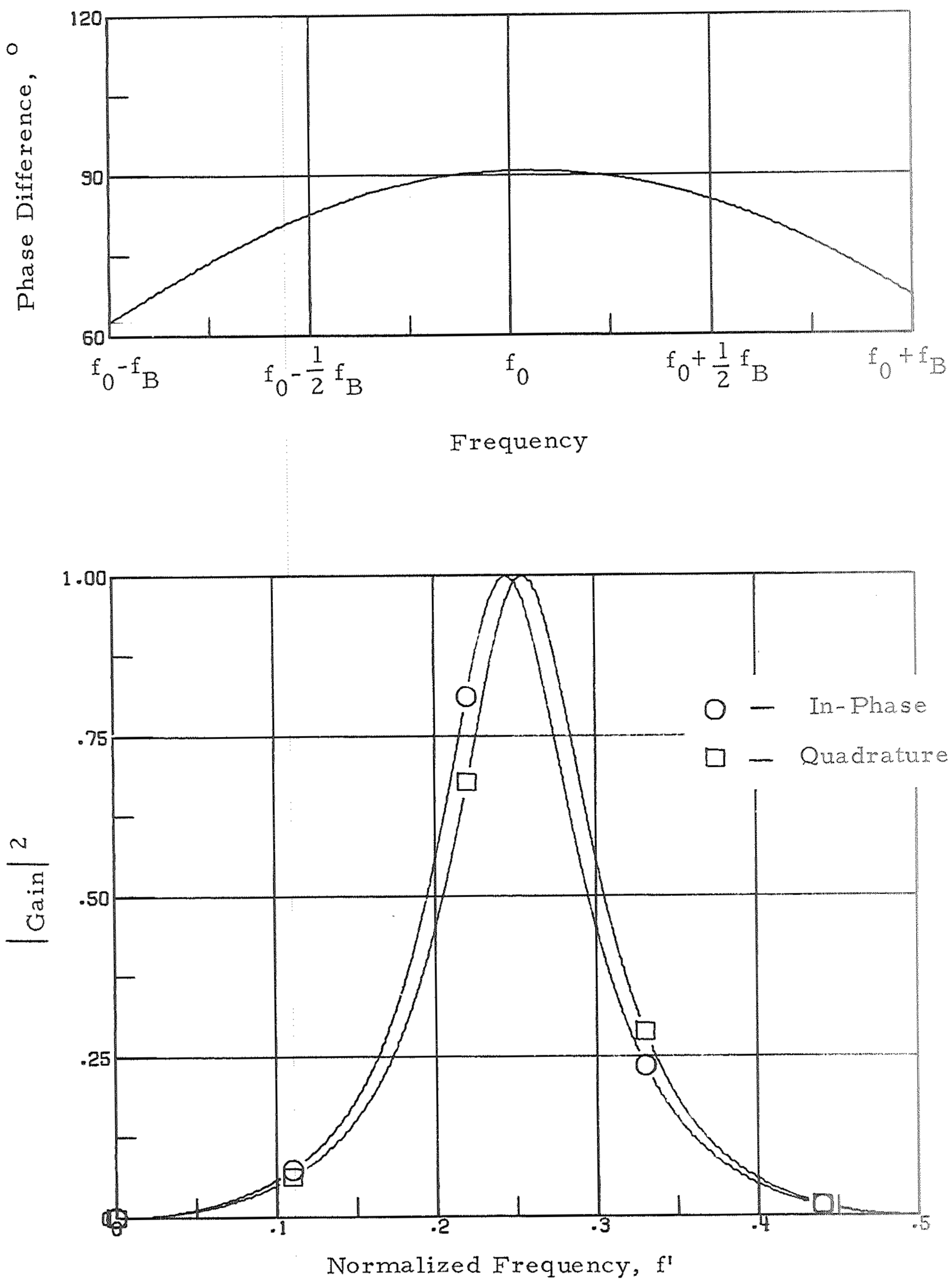


Figure 7.- Analysis band-pass filter design.

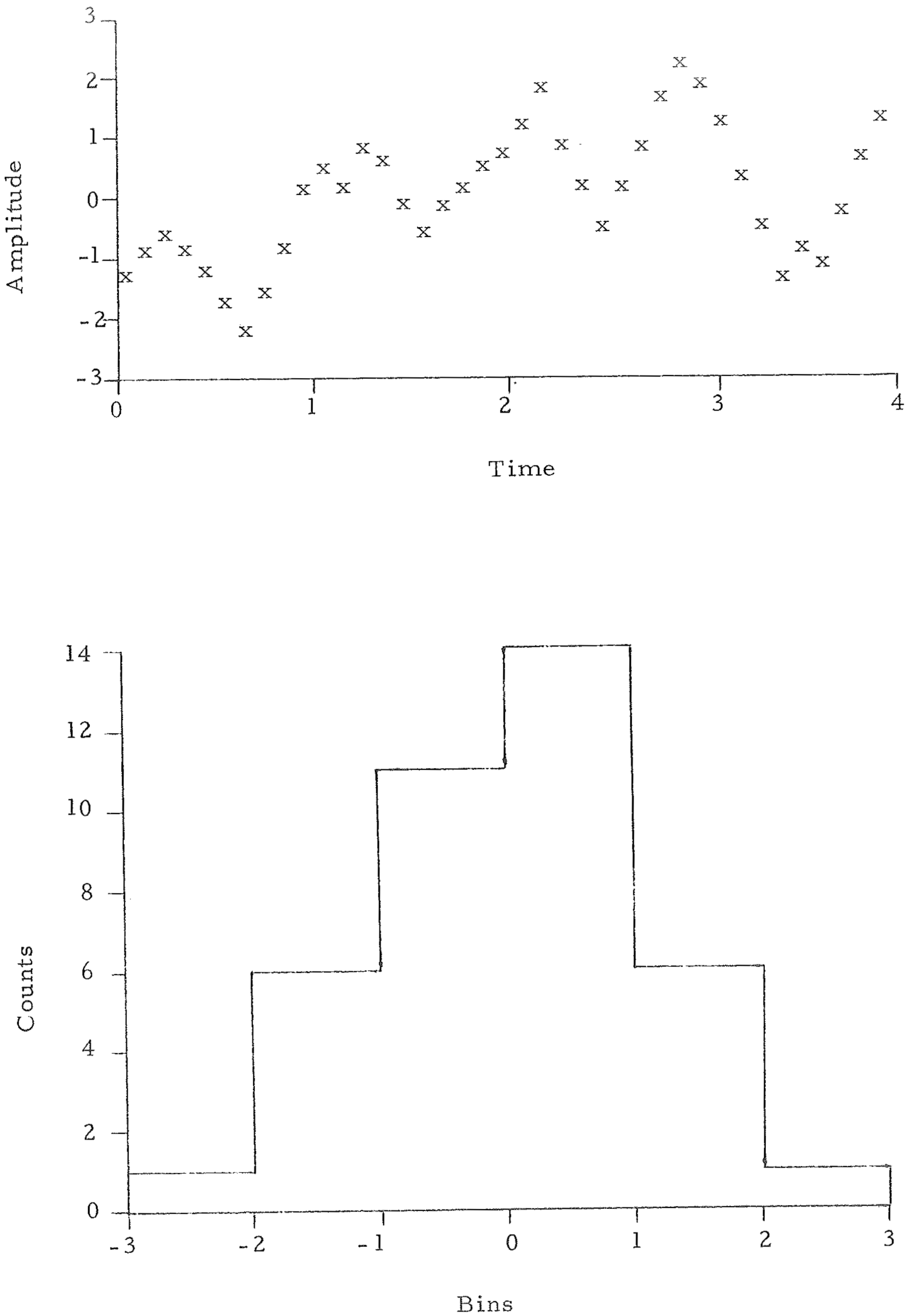
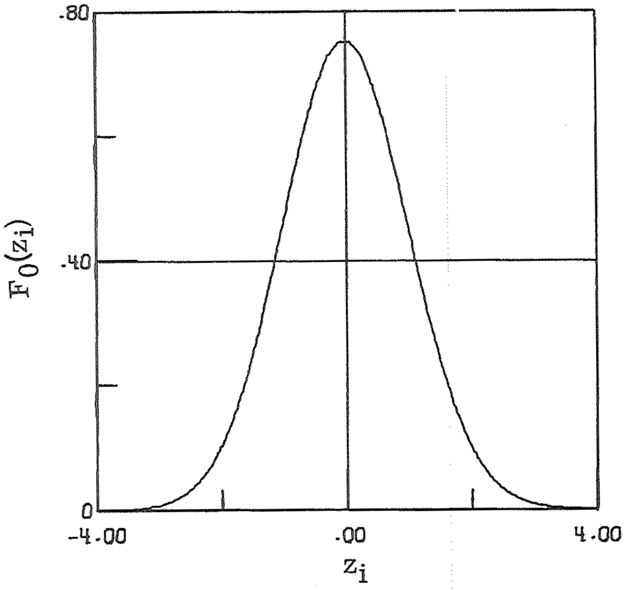
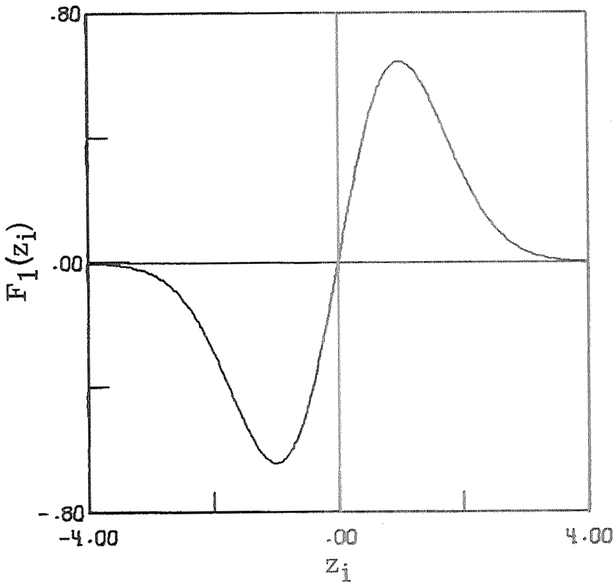


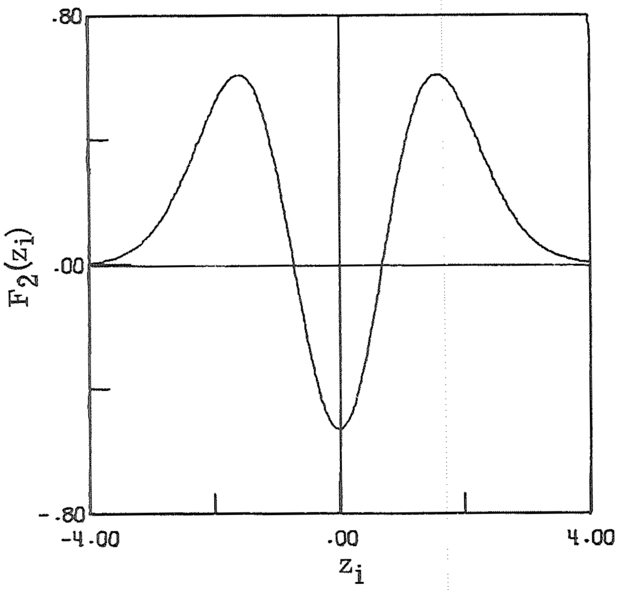
Figure 8.- Example of input data and histogram.



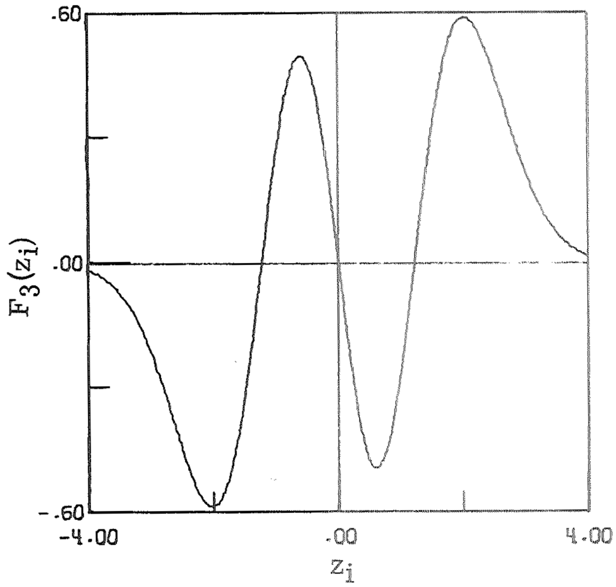
(a) Zero term.



(b) First term.

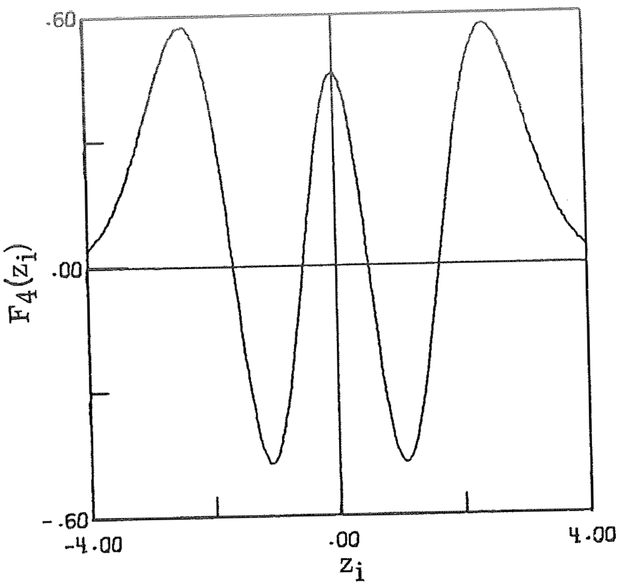


(c) Second term.

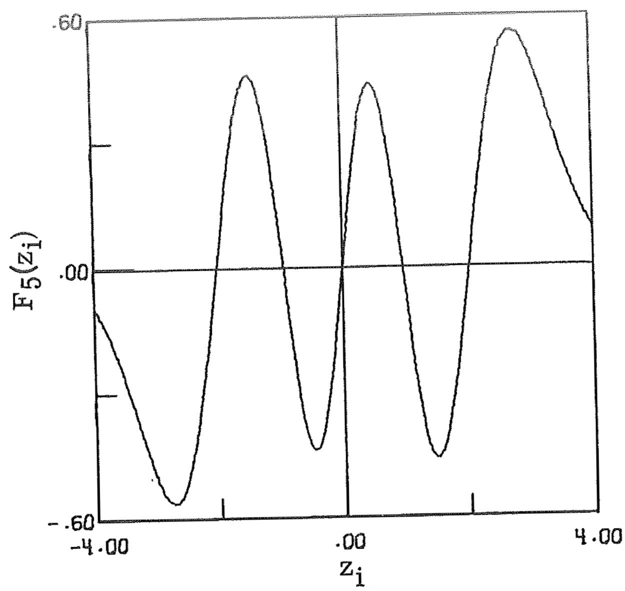


(d) Third term.

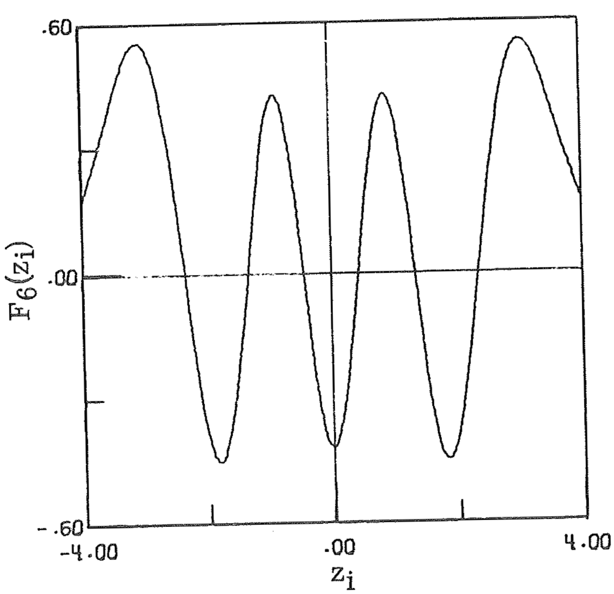
Figure 9.- Hermite functions.



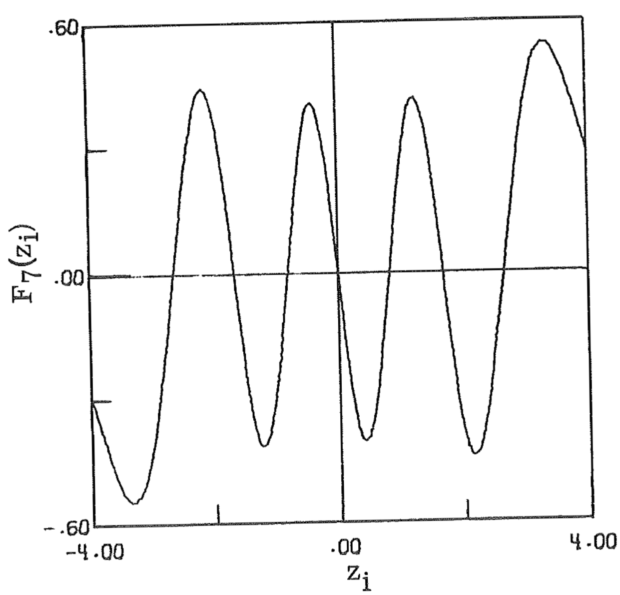
(e) Fourth term.



(f) Fifth term.

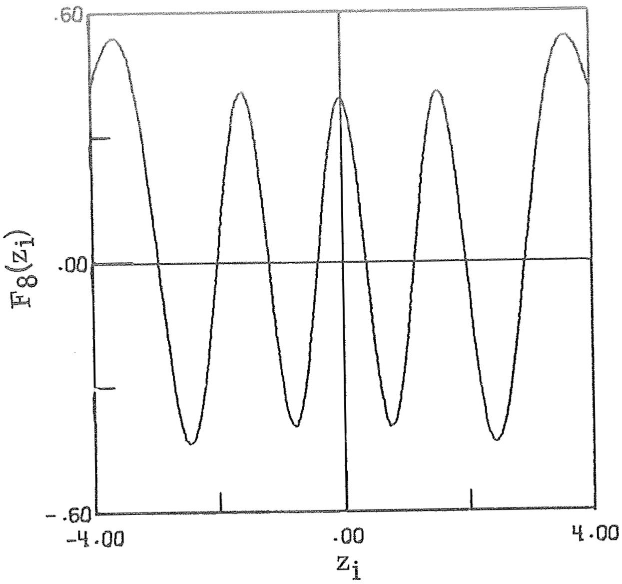


(g) Sixth term.

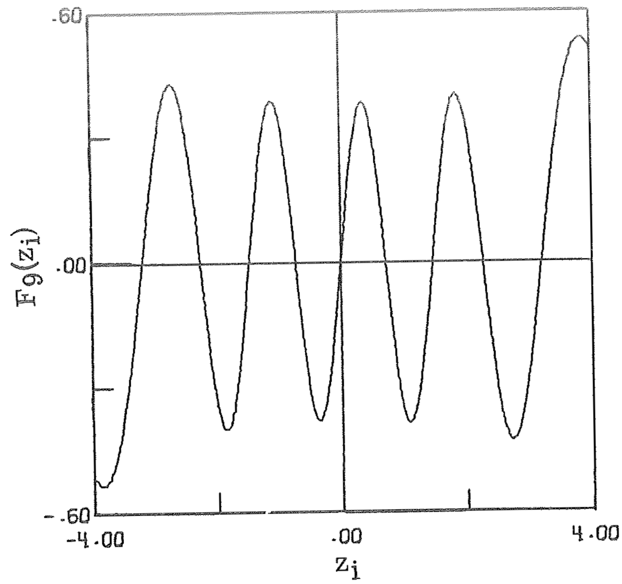


(h) Seventh term.

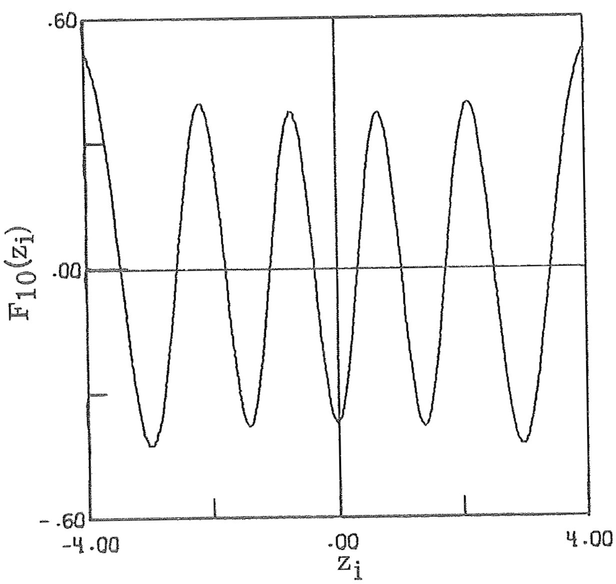
Figure 9.- Continued.



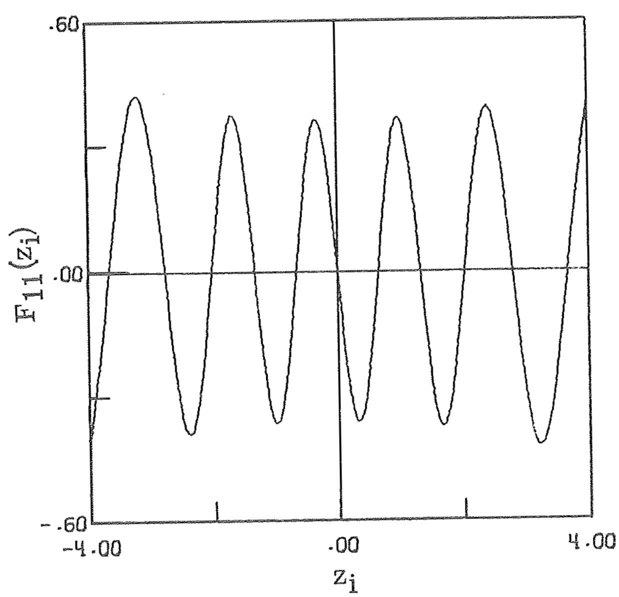
(i) Eighth term.



(j) Ninth term.

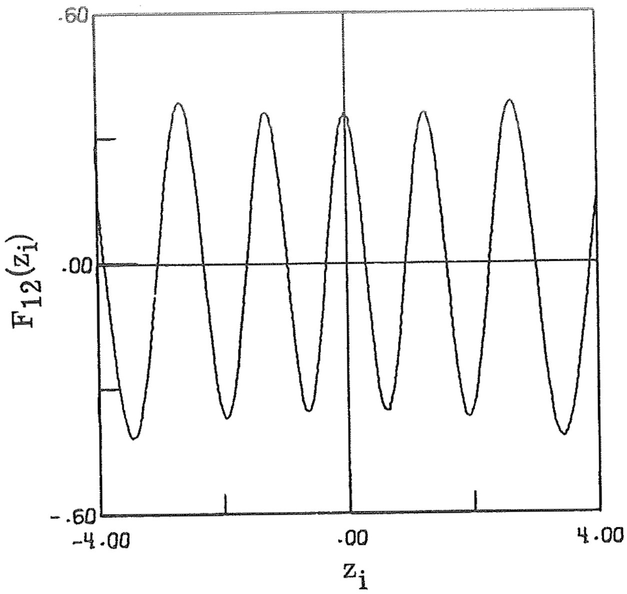


(k) Tenth term.

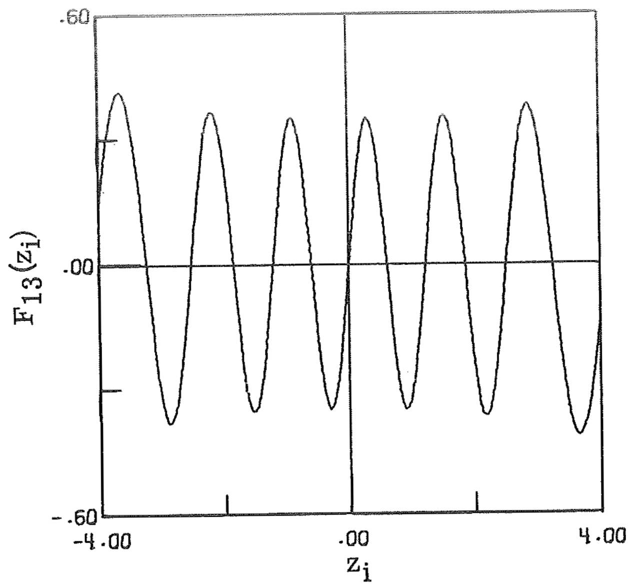


(l) Eleventh term.

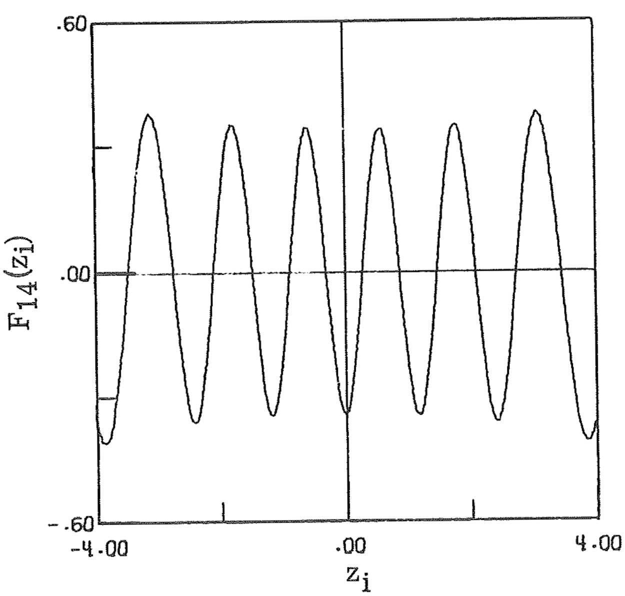
Figure 9.- Continued.



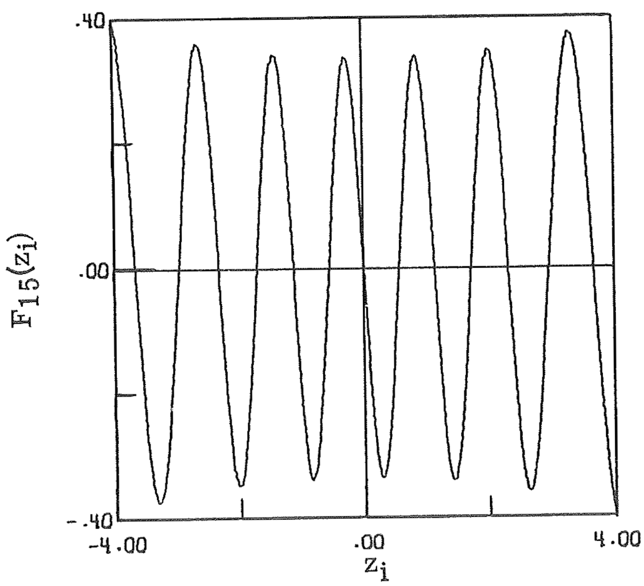
(m) Twelfth term.



(n) Thirteenth term.

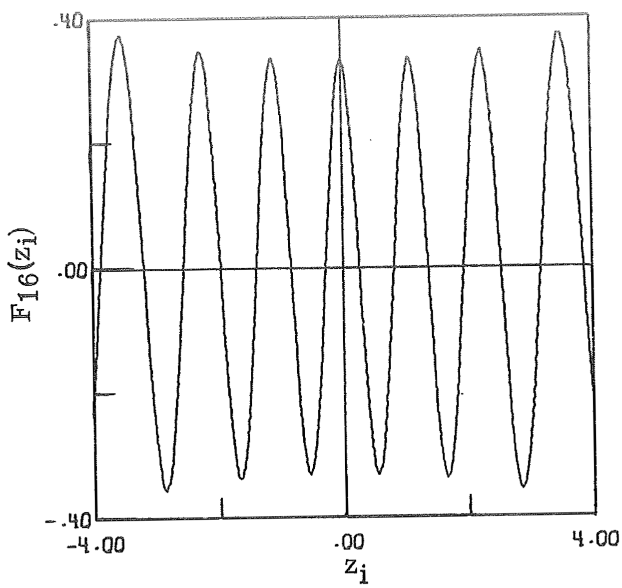


(o) Fourteenth term.

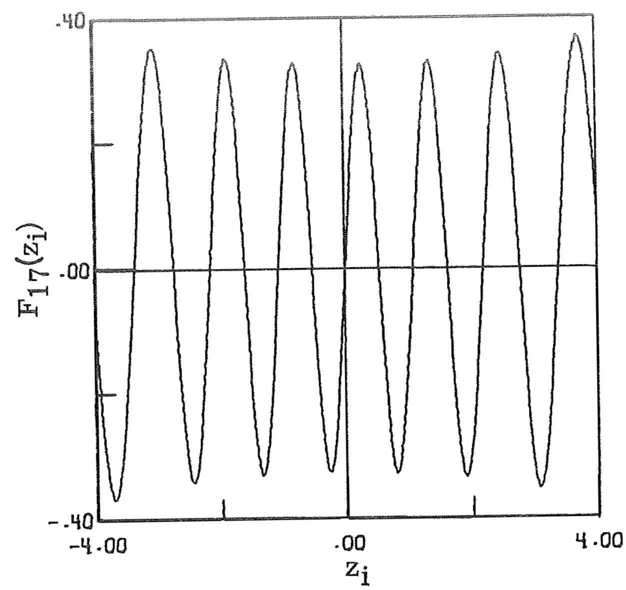


(p) Fifteenth term.

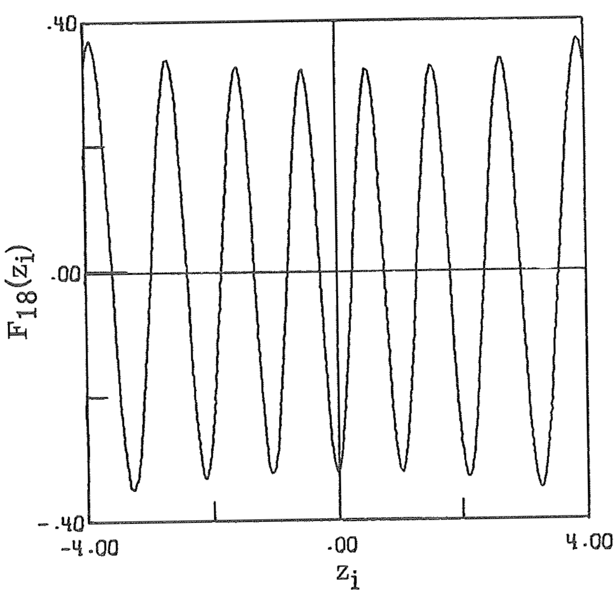
Figure 9.- Continued.



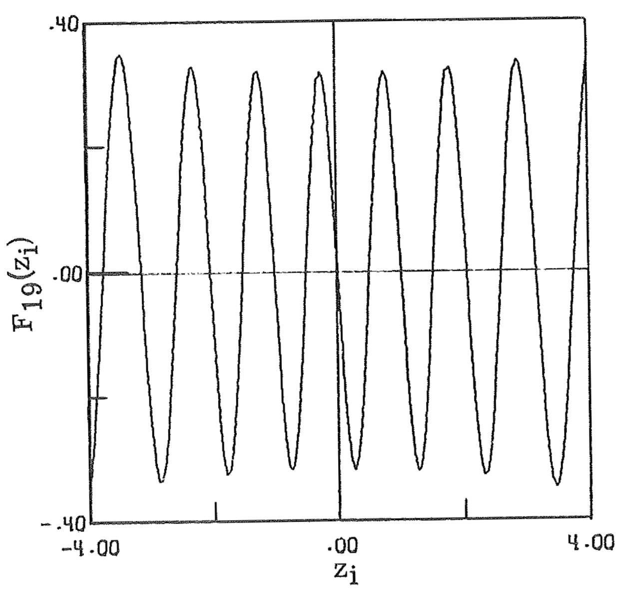
(q) Sixteenth term.



(r) Seventeenth term.

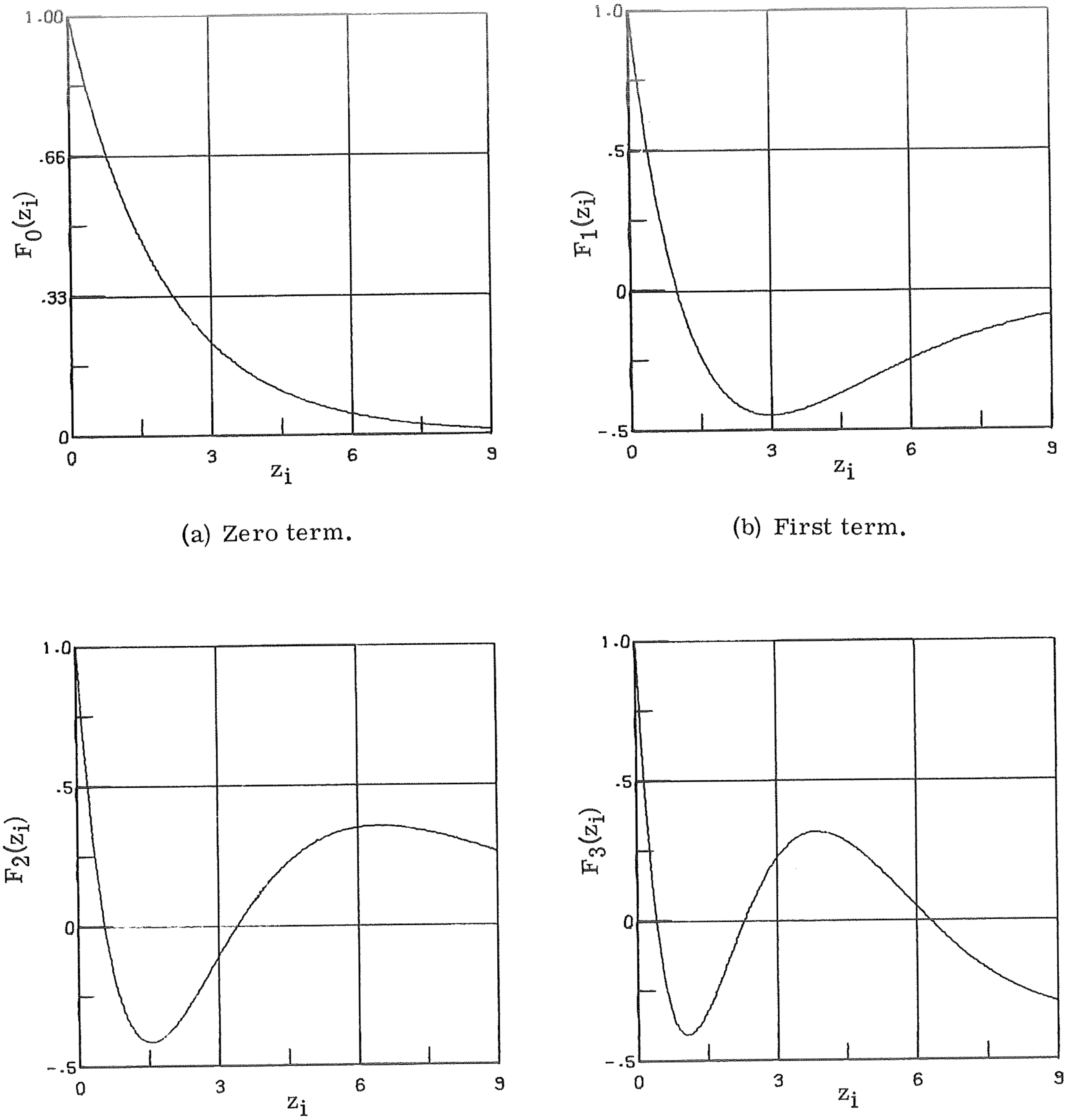


(s) Eighteenth term.



(t) Nineteenth term.

Figure 9.- Concluded.



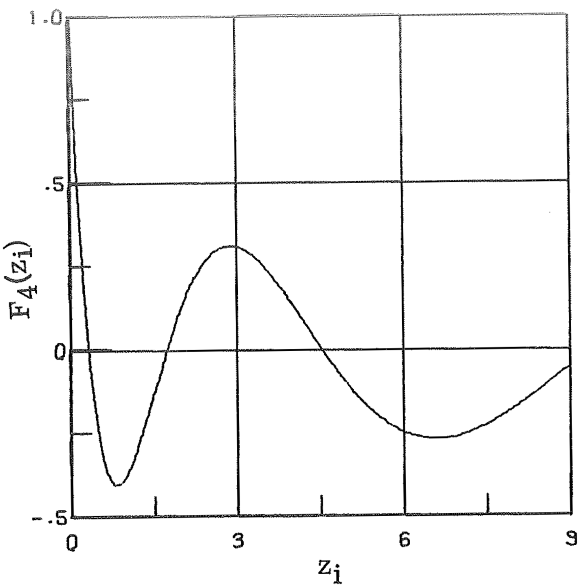
(a) Zero term.

(b) First term.

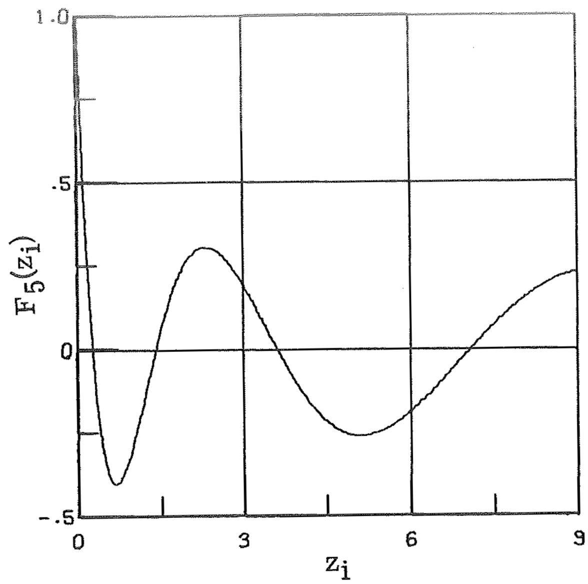
(c) Second term.

(d) Third term.

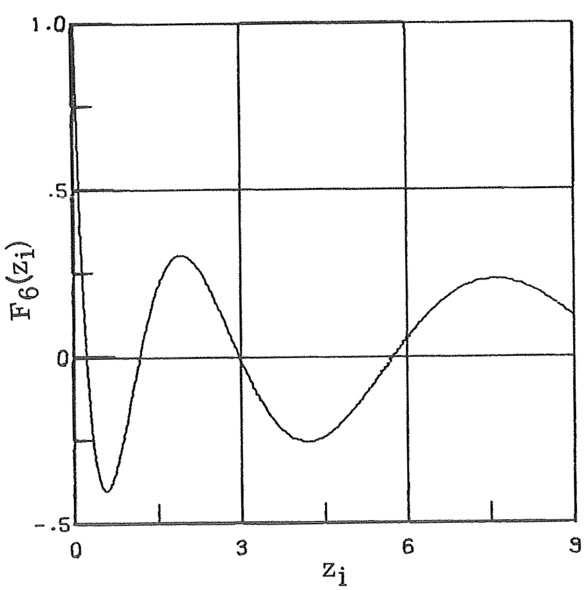
Figure 10.- Laguerre functions.



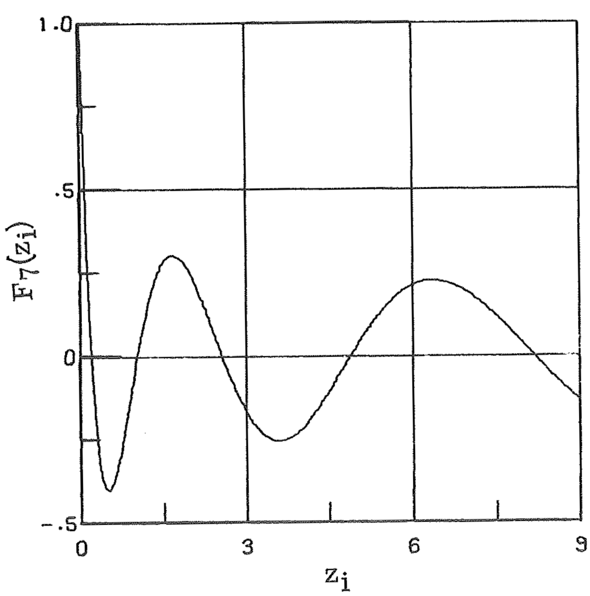
(e) Fourth term.



(f) Fifth term.

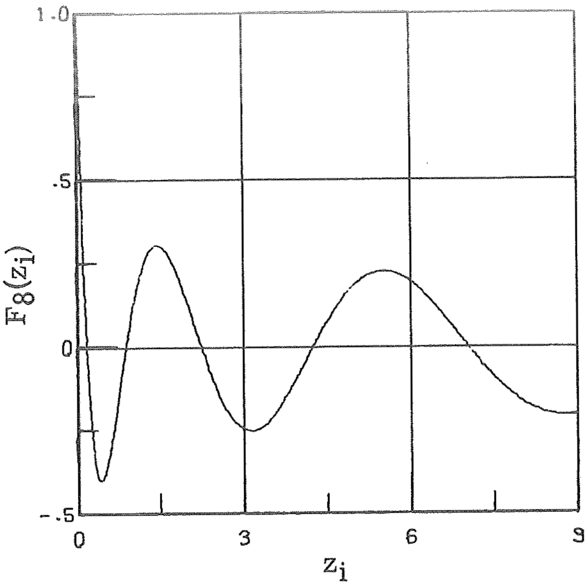


(g) Sixth term.

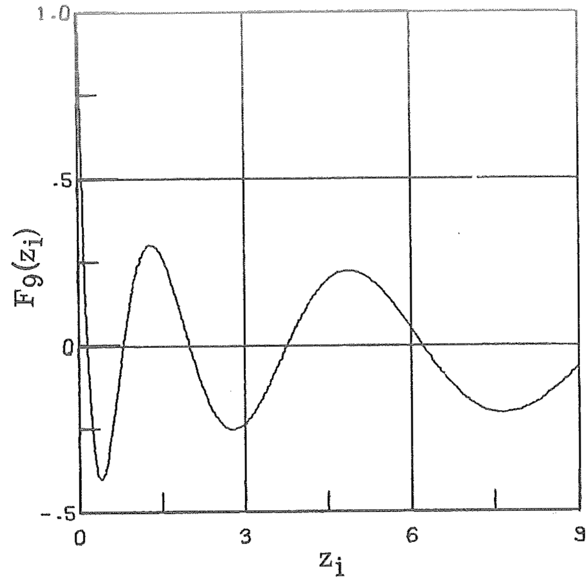


(h) Seventh term.

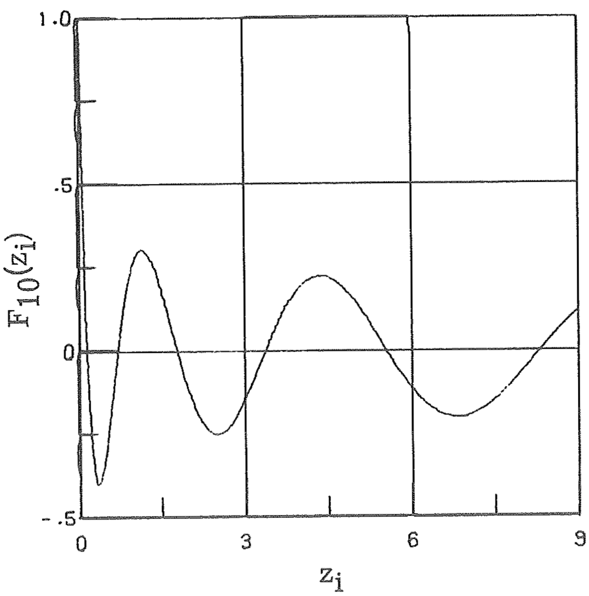
Figure 10.- Continued.



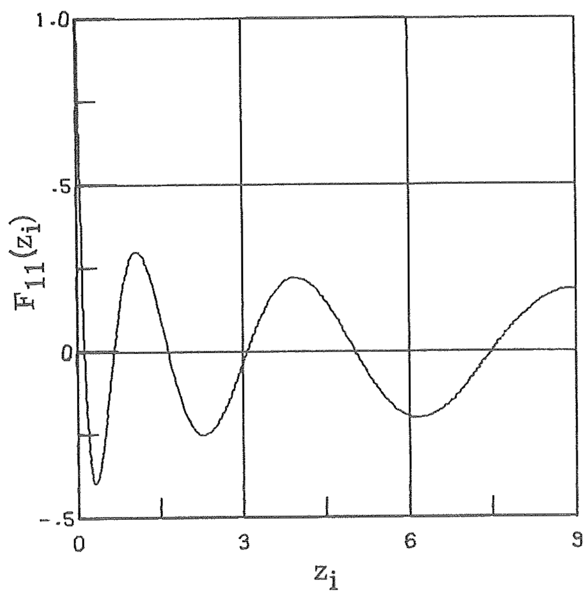
(i) Eighth term.



(j) Ninth term.

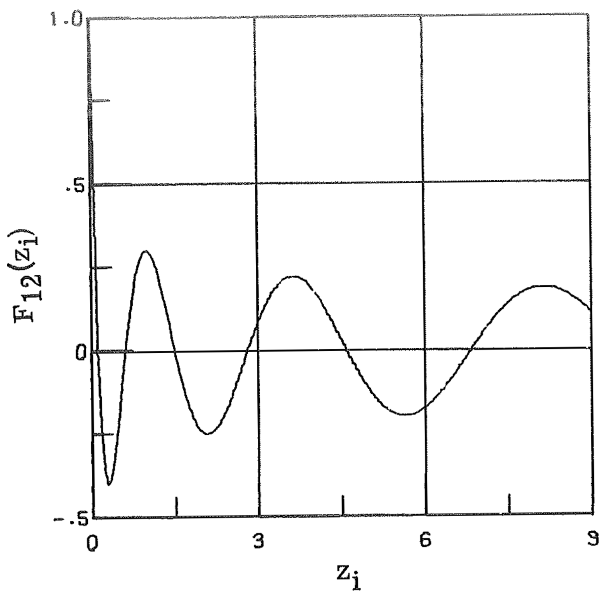


(k) Tenth term.

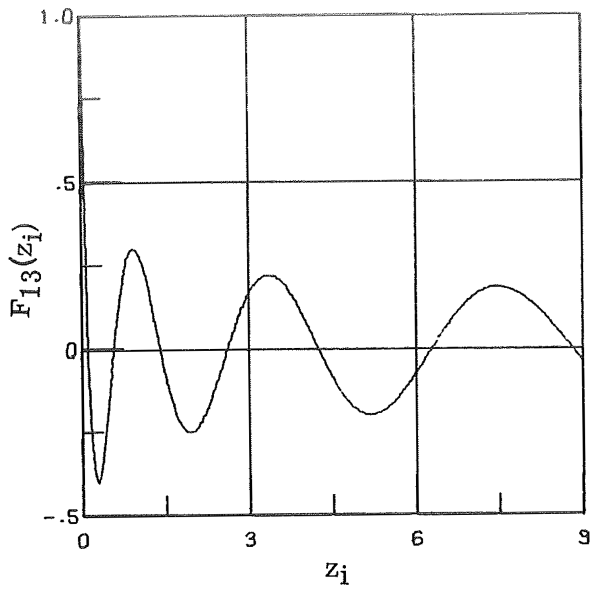


(l) Eleventh term.

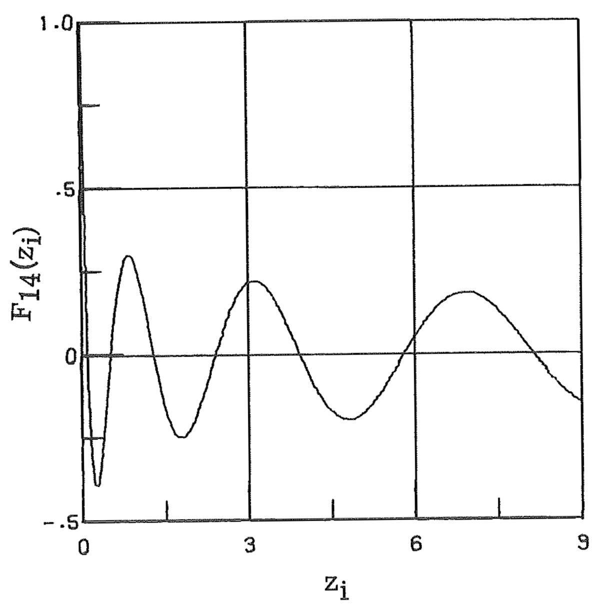
Figure 10.- Continued.



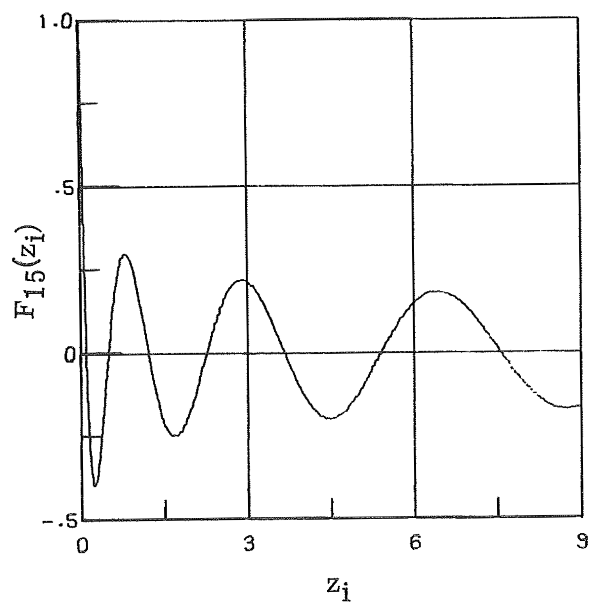
(m) Twelfth term.



(n) Thirteenth term.

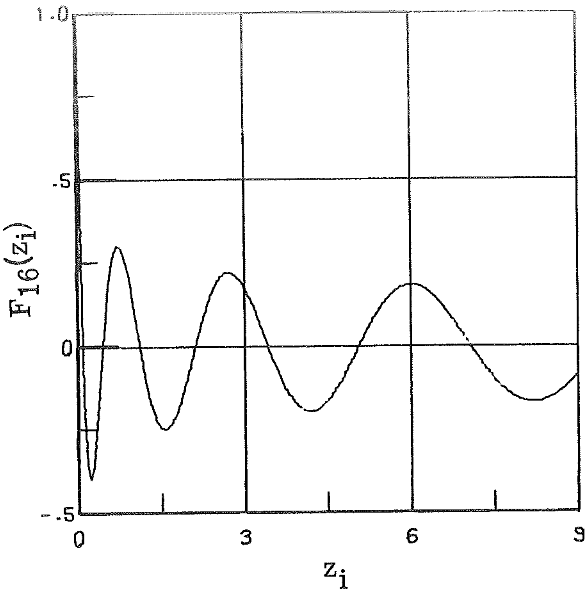


(o) Fourteenth term.

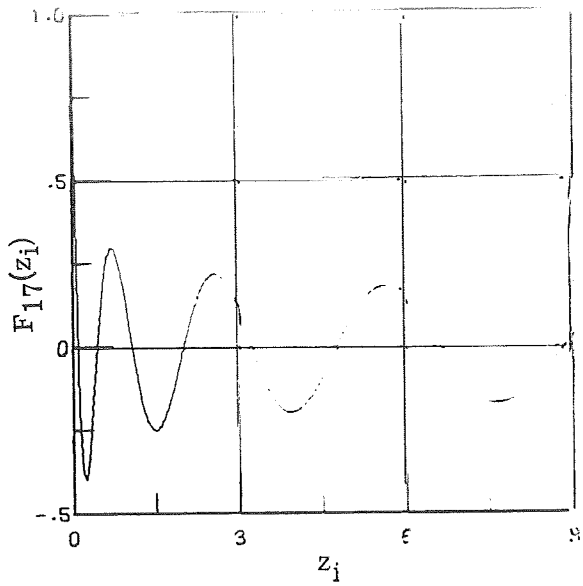


(p) Fifteenth term.

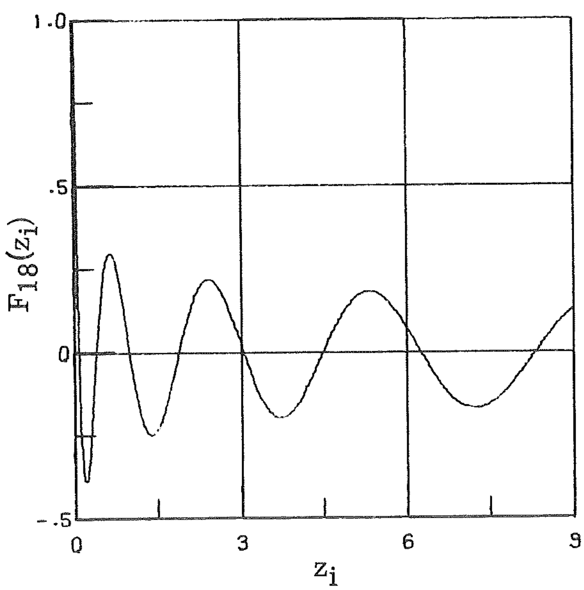
Figure 10.- Continued.



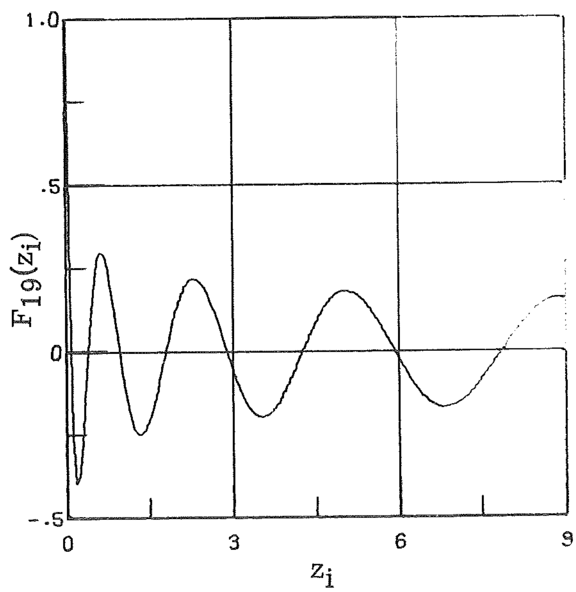
(q) Sixteenth term.



(r) Seventeenth term.



(s) Eighteenth term.



(t) Nineteenth term.

Figure 10.- Concluded.

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